Particle spectrum in the modified NMSSM in the strong Yukawa coupling limit

R.B.Nevzorov and M.A.Trusov

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Abstract

A theoretical analysis of solutions of renormalisation group equations in the MSSM corresponding to the quasi-fixed point conditions shows that the mass of the lightest Higgs boson in this case does not exceed 94 ± 5 GeV. It means that a substantial part of the parameter space of the MSSM is practically excluded by existing experimental data from LEP II. In the NMSSM the upper bound on the lightest Higgs boson mass reaches its maximum in the strong Yukawa coupling regime, when Yukawa constants are considerably larger the gauge ones on the Grand Unification scale. In this paper a particle spectrum in a simple modification of NMSSM which leads to a self-consistent solution in the considered region of the parameter space is studied. This model allows one to get $m_h \sim 125$ GeV even for comparatively low values of $\tan\beta \geq 1.9$. For an analysis of the Higgs boson spectrum and neutralino spectrum a method for diagonalisation of mass matrices proposed formerly is used. The mass of the lightest Higgs boson in this model does not exceed 130.5 ± 3.5 GeV.

1 Introduction

The search for the Higgs boson remains one of the top priorities for existing accelerators as well as for those still at the design stage. This is because this boson plays a key role in the Standard Model which describes all currently available experimental data with a high degree of accuracy. As a result of the spontaneous symmetry breaking $SU(2) \otimes U(1)$ the Higgs scalar acquires a nonzero vacuum expectation value without destroying the Lorentz invariance, and generates the masses of all fermions and vector bosons. An analysis of the experimental data using the Standard Model has shown that there is a 95% probability that its mass will not exceed 210 GeV [1]. At the same time, assuming that there are no new fields and interactions and also no Landau pole in the solution of the renormalisation group equations for the self-action constant of Higgs fields up to the scale $M_{\rm Pl} \approx 2.4 \times 10^{18}$ GeV, we can show that $m_h < 180$ GeV [2], [3]. In this case, physical vacuum is only stable provided that the mass of the Higgs boson is greater than 135 GeV [2]-[6]. However, it should be noted that this simplified model does not lead to unification of the gauge constants [7] and a solution of the hierarchy problem [8]. As a result, the construction of a realistic theory which combines all the fields and interactions is extremely difficult in this case.

Unification of the gauge constants occurs naturally on the scale $M_X \approx 3 \times 10^{16}$ GeV within the supersymmetric generalisation of the Standard Model, i.e., the Minimal Supersymmetric Standard Model (MSSM) [7]. In order that all the fundamental fermions acquire mass in the MSSM, not one but two Higgs doublets H_1 and H_2 must be introduced in the theory, each acquiring the nonzero vacuum expectation value v_1 and v_2 where $v^2 = v_1^2 + v_2^2 = (246\ \text{GeV})^2$. The spectrum of the Higgs sector of the MSSM contains four massive states: two CP–even, one CP–odd, and one charged. An important distinguishing feature of the supersymmetric model is the existing of a light Higgs boson in the CP–even sector. The upper bound on its mass is determined to a considerable extent by the value $\tan\beta = v_2/v_1$. In the tree-level approximation the mass of the lightest Higgs boson in the MSSM does not exceed the mass of the Z-boson ($M_Z \approx 91.2$ GeV): $m_h \le M_Z|\cos 2\beta|$ [9]. Allowance for the contribution of loop corrections to the effective interaction potential of the Higgs fields from a t–quark and its superpartners significantly raises the upper bound on its mass:

$$m_h \le \sqrt{M_Z^2 \cos^2 2\beta + \Delta_{11}^{(1)} + \Delta_{11}^{(2)}}\,. \tag{1}$$

Here $\Delta_{11}^{(1)}$ and $\Delta_{11}^{(2)}$ are the one–loop [10] and two–loop [11] corrections, respectively. The values of these corrections are proportional to m_t^4, where m_t is the running mass of t–quark which depends logarithmically on the supersymmetry breaking scale M_S and is almost independent of the choice of $\tan\beta$. In [3], [5], [6] bounds on the mass of the Higgs boson were compared in the Minimal Standard and Supersymmetric models. The upper bound on the mass of the light CP–even Higgs boson in the MSSM increases with increasing $\tan\beta$ and for $\tan\beta \gg 1$ in realistic supersymmetric models with $M_S \le 1000$ GeV reaches 125-128 GeV.

However, a considerable fraction of the solutions of the system of MSSM renormalisation group equations is focused near the infrared quasi-fixed point at $\tan\beta \sim 1$. In the region of parameter space of interest to us ($\tan\beta \ll 50$) the Yukawa constants of a b–quark (h_b) and a τ–lepton (h_τ) are negligible so that an exact analytic solution can be obtained

for the one–loop renormalisation group equations [12]. For the Yukawa constants of a t–quark $h_t(t)$ and the gauge constants $g_i(t)$ its solution has the following form:

$$Y_t(t)=\frac{\dfrac{E(t)}{6F(t)}}{1+\dfrac{1}{6Y_t(0)F(t)}},\qquad \tilde{\alpha}_i(t)=\frac{\tilde{\alpha}_i(0)}{1+b_i\tilde{\alpha}_i(0)t},$$

$$E(t)=\left[\frac{\tilde{\alpha}_3(t)}{\tilde{\alpha}_3(0)}\right]^{16/9}\left[\frac{\tilde{\alpha}_2(t)}{\tilde{\alpha}_2(0)}\right]^{-3}\left[\frac{\tilde{\alpha}_1(t)}{\tilde{\alpha}_1(0)}\right]^{-13/89},\qquad F(t)=\int\limits_0^t E(t')dt', \tag{2}$$

where the index i has values between 1 and 3,

$$b_1=33/5,\quad b_2=1,\quad b_3=-3$$
$$\tilde{\alpha}_i(t)=\left(\frac{g_i(t)}{4\pi}\right)^2,\quad Y_i(t)=\left(\frac{h_t(t)}{4\pi}\right)^2.$$

The variable t is determined by a standard method $t=\ln(M_X^2/q^2)$. The boundary conditions for the renormalisation group equations are usually set at the grand unification scale M_X $(t=0)$ where the values of all three Yukawa constants are the same: $\tilde{\alpha}_1(0)=\tilde{\alpha}_2(0)=\tilde{\alpha}_3(0)=\tilde{\alpha}(0)$. On the electroweak scale where $h_t^2(0)\gg 1$ the second term in the denominator of the expression describing the evolution of $Y_t(t)$ is much smaller than unity and all the solutions are concentrated in a narrow interval near the quasi-fixed point $Y_{\rm QFP}(t)=E(t)/6F(t)$ [13]. In other words in the low-energy range the dependence of $Y_t(t)$ on the initial conditions on the scale M_X disappears. In addition to the Yukawa constant of the t–quark, the corresponding trilinear interaction constant of the scalar fields A_t and the combination of the scalar masses $\mathfrak{M}_t^2=m_Q^2+m_U^2+m_2^2$ also cease to depend on $A_t(0)$ and $\mathfrak{M}_t^2(0)$ as $Y_t(0)$ increases. Then on the electroweak scale near the infrared quasi-fixed point $A_t(t)$ and $\mathfrak{M}_t^2(t)$ are only expressed in terms of the gaugino mass on the Grand Unification scale. Formally this type of solution can be obtained if $Y_t(0)$ is made to go to infinity. Deviations from this solution are determined by ratio $1/6F(t)Y_t(0)$ which is of the order of $1/10h_t^2(0)$ on the electroweak scale.

The properties of the solutions of the system of MSSM renormalisation group equations and also the particle spectrum near the infrared quasi-fixed point for $\tan\beta\sim 1$ have been studied by many authors [14], [15]. Recent investigations [15]-[17] have shown that for solutions $Y_t(t)$ corresponding to the quasi-fixed point regime the value of $\tan\beta$ is between 1.3 and 1.8. These comparatively low values of $\tan\beta$ yield significantly more stringent bounds on the mass of the lightest Higgs boson. The weak dependence of the soft supersymmetry breaking parameters $A_t(t)$ and $\mathfrak{M}_t^2(t)$ on the boundary conditions near the quasi-fixed point means that the upper bound on its mass can be calculated fairly accurately. A theoretical analysis made in [15], [16] showed that m_h does not exceed 94 ± 5 GeV. This bound is 25-30 GeV below the absolute upper bound in the Minimal Supersymmetric Model. Since the lower bound on the mass of the Higgs boson from LEP II data is 113 GeV [1], which for the spectrum of heavy supersymmetric particles is the same as the corresponding bound on the mass of the Higgs boson in the Standard Model, a considerable fraction of the solutions which come out to a quasi-fixed point in the MSSM, are almost eliminated by existing experimental data. This provides the stimulus for theoretical analyses of the Higgs sector in more complex supersymmetric models.

The simplest expansion of the MSSM which can conserve the unification of the gauge constants and raise the upper bound on the mass of the lightest Higgs boson is the Nonminimal Supersymmetric Standard Model (NMSSM) [18]-[20]. In addition to the doublets H_1 and H_2, the Higgs sector of this model contains the additional singlet superfield Y relative to the gauge $SU(2) \otimes U(1)$ interactions. The most attractive region of the NMSSM parameter space from the point of view of theoretical analysis is that corresponding to the limit of strong Yukawa coupling when the Yukawa constants on the Grand Unification scale M_X are substantially larger than the gauge constant g_{GUT}. This is the region where the upper bound on the mass of the lightest Higgs boson reaches its maximum, which is several GeVs larger than the corresponding absolute bound in the MSSM. In addition, in this particular case it is possible to select the interaction constants so as to achieve the unification of the Yukawa constants of a b–quark and a τ–lepton on the scale M_X [21], [22] which usually occurs in GUTs [23].

However, the mass of the lightest Higgs boson in the NMSSM differs substantially from its upper bound [24]. In this connection, the present paper examines a very simple model in which m_h reaches its upper theoretical bound for a specific choice of fundamental parameters. This bound model is obtained by modifying the Nonminimal Supersymmetric Model and yields a self-consistent solution in the strong Yukawa coupling regime where, even for comparatively low values of $\tan\beta \leq 1.9$, the mass of the lightest Higgs boson in the modified NMSSM may reach 125-127 GeV. Although the parameter space of this model is enlarged considerably, the theory does not lose its predictive capacity. The proposed model is used to study characteristics of the spectrum of superpartners of observable particles and Higgs bosons. The mass of the lightest Higgs boson in this model does not exceed 130.5 ± 3.5 GeV.

This bound on the mass of the lightest Higgs boson is not the absolute upper bound in supersymmetric models. For instance, it was shown in [25] that by introduction four or five additional $5 + \bar{5}$ multiplets of matter, the upper bound on m_h in the NMSSM is increased up to 155 GeV. Recently the upper bound on the mass of the lightest Higgs boson has been actively discussed using more complex expansion of MSSM [26], [28]. In particular, in addition to the singlet it is also possible to introduce several $SU(2)$ triplets into the Higgs sector of supersymmetric models. Their appearance destroys the unification of the gauge constants at high energies. In order to reconstruct this, in addition to triplets we also need to add several multiplets of matter which carry colour charge in the $SU(3)$ group but do not participate in $SU(2) \otimes U(1)$ interactions, for example four $3 + \bar{3}$. A numerical analysis made in [27] shows that unification of the gauge constants then occurs on the scale $\tilde{M}_X \sim 10^{17}$ GeV and the mass of the lightest Higgs boson does not exceed 190 GeV. The existence of a fourth generation of particles in the MSSM [28], which is extremely problematical from the point of view of the known experimental data, also leads to an appreciable increase in the upper bound on m_h. Consequently, an increase in the upper bound on the mass of the lightest Higgs boson in the supersymmetric models is usually accompanied by a substantial increase in the number of particles in the models which may be counted as a serious disadvantage of this type of model. In the present study, unlike those noted above [25], [28], we examine the dependence of m_h and the particle spectrum on the fundamental parameters of the modified NMSSM in the strong Yukawa coupling regime.

2 NMSSM parameters and their renormalisation in the strong Yukawa coupling regime

By definition the superpotential of the Nonminimal Supersymmetric Model is invariant with respect to the discrete transformations $y'_\alpha = e^{2\pi i/3} y_\alpha$ of the Z_3 group [19] which means that we can avoid the problem of the μ-term in supergravity models. Z_3-symmetry usually occurs in string models in which all the fields of the observable sector remain massless in the exact supersymmetric limit. In addition to observable superfields y_α, supergravity theories also contain a hidden sector in which local supersymmetry is broken. In modern supergravity theories this sector includes singlet dilaton S and moduli T_m fields with respect to gauge interactions. These fields always appear in four-dimensional theory and they occur as a result of the compactification of additional dimensions. The vacuum-averaged dilaton and moduli fields determine the values of the gauge constants on the Grand Unification scale and also the dimensions and shape of compacted space. The superpotential in supergravity models is usually represented as an expansion in terms of superfields of the observable sector [29]:

$$W = \hat{W}_0(S, T_m) + \frac{1}{2}\mu_{\alpha\beta}(S, T_m) y_\alpha y_\beta + \frac{1}{6} h_{\alpha\beta\gamma}(S, T_m) y_\alpha y_\beta y_\gamma, \tag{3}$$

where $\hat{W}_0(S, T_m)$ is the superpotential of the hidden sector. In expression (3) summation is performed over the recurrent greek subscripts. The requirements for conservation of R-parity [20] and gauge invariance have the result that the single parameter μ is retained in the MSSM which corresponds to the term $\mu(H_1, H_2)$ in the superpotential (3). However, the expansion (3) assumes that this fundamental parameter should be of the order of $M_{\rm Pl}$ since this scale is the only dimensional parameter characterising the hidden (gravity) sector of the theory. In this case, however, the Higgs bosons H_1 and H_2 acquire an enormous mass $m^2_{H_1,H_2} \sim \mu^2 \sim M^2_{\rm Pl}$ and no breaking of $SU(2) \otimes U(1)$ symmetry occurs. In the NMSSM the term μ in the superpotential (3) is not invariant with respect to discrete transformations of the Z_3 group and for this reason should be eliminated from the analysis ($\mu = 0$). As a result of the multiplicative nature of the renormalisation of this parameter, the term $\mu(q)$ remains zero on any scale $q \le M_X \div M_{\rm Pl}$. However, the absence of mixing of the Higgs doublets on electroweak scale has the result that H_1 acquires no vacuum expectation value as a result of the spontaneous symmetry breaking and d–type quarks and charged leptons remain massless. In order to ensure that all quarks and charged leptons acquire nonzero masses, an additional singlet superfield Y with respect to gauge $SU(2) \otimes U(1)$ transformations is introduced in the NMSSM. The superpotential of the Higgs sector of the Nonminimal Supersymmetric Model [18]-[20] has the following form:

$$W_h = \lambda Y (H_1 H_2) + \frac{\varkappa}{3} Y^3. \tag{4}$$

As a result of the spontaneous breaking of $SU(2) \otimes U(1)$ symmetry, the field Y acquires a vacuum expectation value ($\langle Y \rangle = y/\sqrt{2}$) and the effective μ-term ($\mu = \lambda y/\sqrt{2}$) is generated.

In addition to the Yukawa constants λ and $\varkappa$, and also the Standard Model constants, the Nonminimal Supersymmetric Model contains a large number of unknown parameters. These are the so-called soft supersymmetry breaking parameters which are required

to obtain an acceptable spectrum of superpartners of observable particles form the phenomenological point of view. The hypothesis on the universal nature of these constants on the Grand Unification scale allows us to reduce their number in the NMSSM to three: the mass of all the scalar particles m_0, the gaugino mass $M_{1/2}$, and the trilinear interaction constant of the scalar fields A. In order to avoid strong violation of CP–parity and also spontaneous breaking of gauge symmetry at high energies ($M_{\rm Pl} \gg E \gg m_t$) as a result of which the scalar superpartners of leptons and quarks would require nonzero vacuum expectation values, the complex phases of the soft supersymmetry breaking parameters are assumed to be zero and only positive values of m_0^2 are considered. Naturally universal supersymmetry breaking parameters appear in the minimal supergravity model [31] and also in various string models [29], [32]. In the low-energy region the hypothesis of universal fundamental parameters allows to avoid the appearance of neutral currents with flavour changes and can simplify the analysis of the particle spectrum as far as possible. The fundamental parameters thus determined on the Grand Unification scale should be considered as boundary conditions for the system of renormalisation group equations which describes the evolution of these constants as far as the electroweak scale or the supersymmetry breaking scale. The complete system of the renormalisation group equations of the Nonminimal Supersymmetric Model can be found in [33], [34]. These experimental data impose various constraints on the NMSSM parameter space which were analysed in [35], [36].

The introduction of the neutral field Y in the NMSSM potential leads to the appearance of a corresponding F-term in the interaction potential of the Higgs fields. As a consequence, the upper bound on the mass of the lightest Higgs boson is increased:

$$m_h \le \sqrt{\frac{\lambda^2}{2} v^2 \sin^2 2\beta + M_Z^2 \cos^2 2\beta + \Delta_{11}^{(1)} + \Delta_{11}^{(2)}}. \tag{5}$$

The relationship (5) was obtained in the tree-level approximation ($\Delta_{11} = 0$) in [20]. However, loop corrections to the effective interaction potential of the Higgs fields from the t–quark and its superpartners play a very significant role. In terms of absolute value their contribution to the upper bound on the mass of the Higgs boson remains approximately the same as in the Minimal Supersymmetric Model. When calculating the corrections $\Delta_{11}^{(1)}$ and $\Delta_{11}^{(2)}$ we need to replace the parameter μ by $\lambda y/\sqrt{2}$. Studies of the Higgs sector in the Nonminimal Supersymmetric model and the one–loop corrections to it were reported in [24], [33], [36]-[39]. In [6] the upper bound on the mass of the lightest Higgs boson in the NMSSM was compared with the corresponding bounds on m_h in the Minimal Standard and Supersymmetric Models. The possibility of a spontaneous loss of CP–parity in the Higgs sector of the NMSSM was studied in [39], [40].

It follows from condition (5) that the upper bound on m_h increases as λ increases. Moreover, it only differs substantially from the corresponding bound in the MSSM in the range of small $\tan\beta$. For high values ($\tan\beta \gg 1$) the value of $\sin 2\beta$ tends to zero and the upper bounds on the mass of the lightest Higgs boson in the MSSM and NMSSM are almost the same. The case of small $\tan\beta$ is only achieved for fairly high values of the Yukawa constant of a t–quark h_t on the electroweak scale ($h_t(t_0) \ge 1$ where $t_0 = \ln(M_X^2/m_t^2)$), and $\tan\beta$ decreases with increasing $h_t(t_0)$. However, an analysis of the renormalisation group equations in the NMSSM shows that an increase of the Yukawa constants on the electroweak scale is accompanied by an increase of $h_t(0)$ and $\lambda(0)$ on the Grand Unification scale. It thus becomes obvious that the upper bound on the mass of the lightest Higgs

boson in the Nonminimal Supersymmetric model reaches its maximum on the strong Yukawa coupling limit, i.e., when $h_t(0) \gg g_i(0)$ and $\lambda(0) \gg g_i(0)$.

In our previous two studies [21], [41] we analysed the renormalisation of the NMSSM parameters in the strong Yukawa coupling regime. We showed [21] that as the values of the Yukawa constants on the scale M_X increase, the solutions of the renormalisation group equations on the electroweak scale are pulled towards a quasi-fixed (Hill) line ($\varkappa = 0$) or surface ($\varkappa \neq 0$) in Yukawa constant space, which limit the range of permissible values of h_t, λ and $\varkappa$. Outside this range in the solutions of the renormalisation group equations for $Y_i(t)$, where $Y_t(t) = h_t^2/(4\pi)^2$, $Y_\lambda(t) = \lambda^2/(4\pi)^2$, and $Y_\varkappa(t) = \varkappa^2/(4\pi)^2$, a Landau pole appears below the Grand Unification scale and perturbation theory can not be applied when $q^2 \sim M_X^2$. Along the Hill line or surface the values of $Y_i(t)$ are distributed nonuniformly. As $Y_i(0)$ increases , the region in which the solutions of the renormalisation group equations are concentrated on the electroweak scale in the strong Yukawa coupling regime becomes narrower and in the limit $Y_i(0) \to \infty$ all the solutions are focused near the quasi-fixed points. These points are formed as a result of intersection of the Hill line or surface with the infrared fixed (invariant) line. This line connects the stable fixed point in the strong Yukawa coupling regime [42] with the infrared stable fixed point of the system of NMSSM renormalisation group equations [43]. The invariant lines their properties in the Minimal Standard and Supersymmetric Models were studied in detail in [44].

As with increasing $Y_t(0)$ the Yukawa constants approach the quasi-fixed points, corresponding solutions for the trilinear constants $A_i(t)$ and the combinations of scalar particle masses $\mathfrak{M}_i^2(t)$

$$
\begin{aligned}
\mathfrak{M}_t^2 &= m_Q^2 + m_U^2 + m_2^2, \\
\mathfrak{M}_\lambda^2 &= m_2^2 + m_1^2 + m_y^2, \\
\mathfrak{M}_\varkappa^2 &= 3m_y^2,
\end{aligned}
$$

cease to depend on their initial values on the scale M_X. If the evolution of the gauge and Yukawa constants is known, the rest of the renormalisation group equations of the NMSSM can be considered as a system of linear equations for the soft symmetry breaking parameters. In order to solve this system of equations we first need to integrate the equations for the gaugino masses and for the trilinear interaction constants of the scalar fields $A_i(t)$ and then use the results to calculate $\mathfrak{M}_i^2(t)$. Since the system of differential equations for $A_i(t)$ and $\mathfrak{M}_i^2(t)$ is linear, under universal boundary conditions we can obtain the dependence of the soft supersymmetry breaking parameters on the electroweak scale on A, $M_{1/2}$, and m_0^2 [45], [46]:

$$
\begin{aligned}
A_i(t) &= e_i(t)A + f_i(t)M_{1/2}, \\
\mathfrak{M}_i^2(t) &= a_i(t)m_0^2 + b_i(t)M_{1/2}^2 + c_i(t)AM_{1/2} + d_i(t)A^2.
\end{aligned}
\tag{6}
$$

The functions $e_i(t)$, $f_i(t)$, $a_i(t)$, $b_i(t)$, $c_i(t)$, and $d_i(t)$ remain unknown since no analytic solution of the complete system of NMSSM renormalisation group equations exists. It was shown in [41] that as the quasi-fixed points are approached, the values of the functions $e_i(t_0)$, $a_i(t_0)$, $c_i(t_0)$, and $d_i(t_0)$ tend to zero whereas for $Y_i(0) \to \infty$ all $A_i(t)$ are proportional to $M_{1/2}$ and all $\mathfrak{M}_i^2(t) \propto M_{1/2}^2$. the weak dependence of $A_i(t)$ and $\mathfrak{M}_i^2(t)$ in the strong Yukawa coupling regime on the initial conditions has the result that the solutions of the renormalisation group equations for trilinear interaction constants and

combinations of scalar particle masses and also the solutions for $Y_i(t)$ are focused on the electroweak scale near the quasi-fixed points. In general under nonuniversal boundary conditions the solutions for $A_i(t)$ and $\mathfrak{M}_i^2(t)$ are grouped near certain lines ($\varkappa = 0$) or planes ($\varkappa \neq 0$) in the soft supersymmetry breaking parameter space. These lines and planes are almost perpendicular to the axes A_t and $\mathfrak{M}_t^2$ whereas the planes in the spaces $(A_t.A_\lambda, A_\varkappa)$ and $(\mathfrak{M}_t^2, \mathfrak{M}_\lambda^2, \mathfrak{M}_\varkappa^2)$ are also almost parallel to the axes $A_\varkappa$ and $\mathfrak{M}_\varkappa^2$. Along these lines and planes as $Y_i(0)$ increases, the trilinear interaction constants and combinations of scalar particle masses go to quasi-fixed points.

3 Choice of model

The soft supersymmetry breaking parameters play a key role in an analysis of the particle spectrum in modern supersymmetric models. They destroy the Bose–Fermi degeneracy of the spectrum in supersymmetric theories so that the superpartners of observable particles are substantially heavier than quarks and leptons. However, it should be noted that a study of the particle spectrum in the NMSSM is considerably more complex than a study of this spectrum in the MSSM for $\tan\beta \sim 1$ since two new Yukawa constants λ and $\varkappa$ appear in the nonminimal supersymmetric model for which the boundary conditions are unknown. In turn, the renormalisation of the trilinear interaction constants and the scalar particle masses, i.e, the values of the functions $e_i(t_0)$, $f_i(t_0)$, $a_i(t_0)$, $b_i(t_0)$, $c_i(t_0)$, and $d_i(t_0)$, where $t_0 = 2\ln(M_X/M_t^{\rm pole})$ depends on the choice of $h_t(0)$, $\lambda(0)$, and $\varkappa(0)$ on the grand unification scale.

The most interesting from the point of view of a theoretical analysis is a study of the spectrum of heavy supersymmetric particles when the scale of the supersymmetry breaking $M_S^2 \gg M_Z^2$. This is primarily because in this limit the contribution of new particles to the electroweak observable ones is negligible (see, for example [47]). As has been noted, the Standard Model highly accurately describes all the existing experimental data. Additional Higgs fields and superpartners of observable particles interacting with vector $W^\pm$ and Z bosons make a nonzero contribution to the electroweak observables. However, for $M_S^2 \gg M_Z^2$ their contribution is suppressed in a power fashion as $(M_Z/M_S)^2$, where any increase in the scale of the supersymmetry breaking leads to convergence of the theoretical predictions for the strong interaction constant $\alpha_s(M_Z)$ which may be obtained assuming unification of the gauge constants [48], with the results of an analysis of the experimental data [49]. In addition, it should be noted that the mass of the lightest Higgs boson which is one of the central objects of investigation in any supersymmetric model reaches its highest value for $M_S \sim 1-3$ TeV.

Unfortunately, in the strong Yukawa coupling regime in the NMSSM with a minimal set of fundamental parameters it is impossible to obtain a self-consistent solution which on the one hand would lead to a spectrum with heavy superparticles and on the other could give a mass of the lightest Higgs boson greater than that in the MSSM. When calculating the particle spectrum, the fundamental parameters A, m_0^2, and $M_{1/2}$ on the scale M_X should be selected so that the derivatives of the interaction potential of the scalar potential of the scalar fields $V(H_1, H_2, Y)$ with respect to the vacuum expectation

values v_1, v_2, and y would be zero at the minimum:

$$\frac{\partial V(v_1,v_2,y)}{\partial v_1}=0, \quad \frac{\partial V(v_1,v_2,y)}{\partial v_2}=0, \quad \frac{\partial V(v_1,v_2,y)}{\partial y}=0. \tag{7}$$

Since the trilinear interaction constants and the scalar particle masses in the strong Yukawa coupling regime are almost independent of A, Eqs.(7) link the vacuum expectation value of the neutral scalar field $\langle Y\rangle$, and the parameters m_0^2 and $M_{1/2}$. The value of $\tan\beta$ is determined using the Yukawa constant of a t–quark on the electroweak scale (see below). Then a spectrum of heavy supersymmetric particles is only achieved when $\lambda/\varkappa \gg 1$. However, in this region of parameter space the value of m_h^2 becomes negative and the physical vacuum is unstable which can be attributed to the strong mixing of the CP–even components of the neutral field Y and the superposition of Higgs doublets $h = H_1\cos\beta + H_2\sin\beta$.

Studies of the particle spectrum in the nonminimal supersymmetric model [33],[45],[46],[50] have shown that a self-consistent nontrivial solution of the system of nonlinear algebraic Eqs.(7) for $|\langle Y\rangle| \le 10$ TeV which determines the position of the minimum of the interaction potential of the scalar fields, only exists for $\lambda^2(t_0), \varkappa^2(t_0) \lesssim 0.1$. In this case a strict correlation exists between the fundamental parameters of the NMSSM. In particular, in order to ensure that spontaneous breaking of $SU(2)\otimes U(1)$ symmetry occurs and the field Y has a nonzero vacuum expectation value of the field, the condition $|A_\varkappa/m_y| \ge 3$ must be satisfied. However, the following inequalities must also be satisfied:

$$\begin{aligned} A_l^2 &\le 3(m_1^2 + m_{E_L}^2 + m_{E_R}^2), \\ A_d^2 &\le 3(m_1^2 + m_{D_L}^2 + m_{D_R}^2), \\ A_u^2 &\le 3(m_2^2 + m_{U_L}^2 + m_{U_R}^2). \end{aligned}$$

Otherwise, the superpartners of leptons and quarks acquire vacuum expectation values [51]. All these constraints have the result that the ratio $|A/m_0|$ varies between 3 and 4. In [52], [53] the particle spectrum in the NMSSM is analysed separately for $\tan\beta = m_t(m_t)/m_b(m_t)$ and under nonuniversal boundary conditions.

The limit $h_t^2 \gg \lambda^2, \varkappa^2$ in the nonminimal supersymmetric model corresponds to the MSSM [33]. For $\varkappa = 0$ the Lagrangian of the Higgs sector of the NMSSM is invariant with respect to the global $SU(2)\otimes U(1)\otimes U(1)$ transformations. As a result of the spontaneous symmetry breaking, only the $U(1)$ symmetry corresponding to electromagnetic interaction remains unbroken, which leads to four massless degrees of freedom. Two of these are eaten by a charged $W^\pm$ boson and one by a Z boson. Ultimately, the spectrum of the nonminimal supersymmetric model for $\varkappa = 0$ contains one physical massless state which corresponds to the CP–odd component of the field Y. For low values $\varkappa^2 \ll \lambda^2 h_t^2$ the mass of the lightest CP–odd boson is nonzero and is proportional to the self-action constant of the neutral superfield Y. If the Yukawa constants $\lambda, \varkappa \sim 10^{-3} - 10^{-4}$, for a certain choice of fundamental parameters the mass of the lightest CP–even Higgs boson may be only a few GeV [33],[46],[54]. The main contribution to its wave function is made by the neutral scalar field Y which makes it very difficult to search for this on existing accelerators and those at the design stage since the interaction constants of this type of Higgs boson with gauge bosons and fermions are small. In this limiting case, the lightest stable supersymmetric particle having R–parity of -1 is usually the superpartner of the neutral scalar field Y [33], [46].

However, unlike the minimal supersymmetric model, the discrete Z_3 symmetry which can avoid problems of the μ–term in the NMSSM has the result that three degenerate vacuums appear in the theory because of the breaking of gauge symmetry. Immediately after a phase transition on the electroweak scale the is filled equally with three degenerate phases. The entire space is then divided into separate regions in each of which a particular case is achieved. The regions are separated by domain walls with the surface energy density $\sigma \sim v^3$. Data from cosmological observations eliminate the existence of domain walls. The domain structure of vacuum in the NMSSM is destroyed if the vacuum degeneracy [55] caused by Z_3 symmetry disappears. It was shown in [56] that breaking of Z_3 symmetry by introducing into the NMSSM Lagrangian nonrenormalisable operators of dimension $d = 5$ which do not break the $SU(2) \otimes U(1)$ symmetry can be used to obtain splitting of initially degenerate vacuums such that the domain walls disappear before the beginning of the nucleosynthesis are ($T \sim 1$ MeV). Although operators of dimension $d = 5$ are suppressed with respect to $M_{\rm Pl}$ in supergravity models, their introduction leads to quadratic divergences in the two–loop approximation, i.e., to the problem of hierarchies. Consequently, linear and quadratic terms with respect to superfields are generated in the superpotential of this theory and the vacuum expectation value of the neutral scalar Y is of the order of 10^{11} GeV.

In order to avoid a vacuum domain structure and obtain a self-consistent solution in the strong Yukawa coupling regime, we need to modify the nonminimal supersymmetric model. The NMSSM can be modified most simply by introducing additional terms in the superpotential of the Higgs sector: $\mu(H_1H_2)$ and $\mu' Y^2$ which are not forbidden by gauge $SU(2) \otimes U(1)$ and R symmetries. The additional bilinear terms in the NMSSM superpotential destroy the Z_3 symmetry and no domain structures appear in this theory since no system of degenerate vacuums exists. The introduction of the parameter μ ensures that it is possible to obtain a spectrum of heavy supersymmetric particles in the strong Yukawa coupling regime in the modified model and for a certain choice of μ' the mass of the lightest Higgs boson reaches its upper bound. In this case the mass of the lightest Higgs boson has its highest value $\varkappa = 0$ since as $\varkappa(t_0)$ increases, the upper bound on its mass is reduced as a result of a decrease in $\lambda(t_0)$. In the limit $\varkappa = 0$ the CP–odd Higgs sector of the modified NMSSM contains no physical massless states since in this case, the global symmetry of the Lagrangian is the same as the local symmetry which eliminates the Yukawa self-action constant of the neutral field Y from the analysis. Assuming that this is zero and neglecting all the Yukawa constants except for λ and h_t, the complete superpotential of the modified NMSSM can be expressed in the following form:

$$W_{\rm NMSSM} = \mu(H_1H_2) + \mu' Y^2 + \lambda Y(H_1H_2) + h_t(H_2Q)U_R^C \, . \tag{8}$$

where U_R^C is the charge-coupled right superfield of a t–quark and Q is a doublet of left superfields of b and t–quarks.

In supergravity models, bilinear terms with respect to the superfields may be generated in the superpotential (8) as a result of the additional term $Z(H_1H_2) + h.c.$ in the Kähler potential [57],[58] or nonrenormalisable interaction of the fields of the observable and hidden sectors. The appearance of nonrenormalisable operators of this type in the superpotential of supergravity models may be attributed to nonperturbative effects (for instance, gaugino condensation) [58],[59]. In addition to the parameters μ and μ', this model also sees the appearance of the corresponding bilinear interaction constants of the

scalar fields B and B' which for a minimal choice of fundamental parameters should be assumed to the equal on the grand unification scale. Thus, the nonminimal supersymmetric model may include seven fundamental parameters in addition to the constants of the Standard Model:

$$\lambda,\ \mu,\ \mu',\ A,\ B,\ m_0,\ M_{1/2}\,.$$

4 Constraints on the parameter space of the modified NMSSM

Despite a substantial expansion of the parameter space, the theory does not lose its predictive capacity. An analysis of the behaviour of the solutions of the NMSSM renormalisation group equations in the strong Yukawa coupling limit for $\varkappa = 0$ showed that for $Y_i(0) \to \infty$ all the solutions are concentrated near the quasi-fixed point:

$$\begin{aligned}
&\rho_t^{\rm QFP}(t_0) = 0.803\,, \quad \rho_{A_t}^{\rm QFP}(t_0) = 1.77\,, \quad\ \ \rho_{\mathfrak{M}_t^2}^{\rm QFP}(t_0) = 6.09\\
&\rho_\lambda^{\rm QFP}(t_0) = 0.224\,, \quad \rho_{A_\lambda}^{\rm QFP}(t_0) = -0.42\,, \quad \rho_{\mathfrak{M}_\lambda^2}^{\rm QFP}(t_0) = -2.28\,,
\end{aligned} \tag{9}$$

where $\rho_t(t) = \dfrac{Y_t(t)}{\tilde{\alpha}_3(t)}$, $\rho_\lambda(t) = \dfrac{Y_\lambda(t)}{\tilde{\alpha}_3(t)}$, $\rho_{A_i}(t) = \dfrac{A_i(t)}{M_{1/2}}$ and $\rho_{\mathfrak{M}_\lambda^2}(t) = \dfrac{\mathfrak{M}_\lambda^2(t)}{M_{1/2}^2}$. Thus, at the first stage of the analysis we fixed the initial values of the Yukawa constants $\lambda^2(0) = h_t^2(0) = 10$ corresponding to the quasi-fixed point regime (9) of the renormalisation group equations and also the supersymmetry breaking scale $M_3(1000\ \text{GeV}) = 1000$ GeV which determines the mass scale of all the supersymmetric particles.

Existing FNAL experimental data from measurements of the mass of a t–quark can uniquely relate $\tan\beta$ to the Yukawa constant h_t of a t–quark. The running mass of a t–quark generated when the $SU(2)\otimes U(1)$ symmetry is broken is directly proportional to $h_t(t_0)$:

$$m_t(M_t^{\rm pole}) = \frac{h_t(M_t^{\rm pole})}{\sqrt{2}} v \sin\beta. \tag{10}$$

However, the value of $m_t(M_t^{\rm pole})$ calculated in the $\bar{M}S$ scheme [60] is equal to $m_t(M_t^{\rm pole}) = 165 \pm 5$ GeV. The inaccuracy in determining the running mass of a t–quark is primarily attributable to the experimental error with which its pole mass is measures ($M_t^{\rm pole} = 174.3 \pm 5.1$ GeV [61]). For each fixed set of boundary conditions $h_t(0)$ and $\lambda(0)$, using renormalisation group equations we can calculate the Yukawa constant of a t–quark on the electroweak scale and then, substituting the value obtained $h_t(t_0)$ into formula (10), we can determine the value of $\tan\beta$. In the infrared quasi-fixed point regime we obtain $\tan\beta \approx 1.88$ for $m_t(M_t^{\rm pole}) = 165$ GeV (Tables 1 and 2).

An additional constant which fixes M_3 can be used to determine one of the supersymmetry breaking parameters $M_{1/2}$. The values of all the other dimensional parameters μ, μ', A, B, and m_0 should be selected so that spontaneous breaking of gauge $SU(2)\otimes U(1)$ symmetry

occurs on the electroweak scale. The complete interaction potential of the Higgs fields in the modified NMSSM can be expressed as the sum:

$$V(H_1,H_2,Y)=\mu_1^2|H_1|^2+\mu_2^2|H_2|^2+\mu_y|Y|^2+\Big[\mu_3^2(H_1H_2)+\mu_4^2Y^2+\lambda A_\lambda Y(H_1H_2)+$$
$$+\lambda\mu' Y^*(H_1H_2)+\lambda\mu Y(|H_1|^2+|H_2|^2)+h.c.\Big]+\lambda^2|(H_1H_2)|^2+\lambda^2Y^2(|H_1|^2+|H_2|^2)+$$
$$+\frac{g'^2}{8}(|H_2|^2-|H_1|^2)^2+\frac{g^2}{8}(H_1^+\boldsymbol{\sigma}H_1+H_2^+\boldsymbol{\sigma}H_2)^2+\Delta V(H_1,H_2,Y)\,, \quad (11)$$

where $\Delta V(H_1,H_2,Y)$ are the one–loop corrections to the effective interaction potential; g and g' are the constants of the gauge $SU(2)$ and $U(1)$ interactions ($g_1=\sqrt{5/3}g'$). The constants μ_i^2 in the interaction potential (11) are related to the soft supersymmetry breaking parameters as follows:

$$\mu_1^2=m_1^2+\mu^2\,,\quad \mu_2^2=m_2^2+\mu^2\,,\quad \mu_y^2=m_y^2+\mu'^2\,,$$
$$\mu_3^2=B\mu\,,\quad \mu_4^2=\frac{1}{2}B'\mu'\,.$$

where

$$m_1^2(M_X)=m_2^2(M_X)=m_y^2(M_X)=m_0^2\,,$$
$$B(M_X)=B'(M_X)=B_0\,.$$

The position of the physical minimum of the interaction potential of the Higgs fields (11) is determined by Eqs.(7). Since the vacuum expectation value v and $\tan\beta$ are known, the system of Eqs.(7) can be used to find μ and B_0. Then, it is convenient to introduce $\mu_{\rm eff}=\mu+\dfrac{\lambda y}{\sqrt{2}}$ instead of μ. After various transformations we obtain:

$$\mu_{\rm eff}^2=\frac{m_1^2-m_2^2\tan^2\beta+\Delta_Z(\mu_{\rm eff})}{\tan^2\beta-1}-\frac{1}{2}M_Z^2$$
$$\left(m_1^2+m_2^2+2\mu_{\rm eff}^2+\frac{\lambda^2}{2}v^2+\Delta_\beta(\mu_{\rm eff})\right)\sin 2\beta=-2\left(B\mu+\frac{\lambda y X_2}{\cos 2\beta}\right) \quad (12)$$
$$y\left(m_y^2+\mu'^2+B'\mu'\right)=\frac{\lambda}{2}v^2X_1-\Delta_y(\mu_{\rm eff})\,,$$

where Δ_i corresponds to the contribution of the one–loop corrections:

$$\Delta_\beta=\frac{2}{v^2\tan 2\beta}\frac{\partial\Delta V}{\partial\beta}+4\frac{\partial\Delta V}{\partial v^2}\,,\quad \Delta_y=\frac{\partial\Delta V}{\partial y}\,,$$
$$\Delta_Z=\frac{1}{\cos^2\beta}\left\{2\frac{\partial V}{\partial v^2}\cos 2\beta-\frac{1}{v^2}\frac{\partial\Delta V}{\partial\beta}\sin 2\beta\right\}\,,$$

and

$$X_1=\frac{1}{\sqrt{2}}\left(2\mu_{\rm eff}+(\mu'+A_\lambda)\sin 2\beta\right)\,,\quad X_2=\frac{1}{\sqrt{2}}\left(\mu'+A_\lambda\right)\cos 2\beta.$$

When calculating the one–loop corrections we shall only take into account the contribution from loops containing a t–quark and its superpartners since their contribution is

dominant. In supersymmetric theories each fermion state with a specific chirality has a scalar superpartner. Thus, a t–quark incorporating left and right chiral components has two scalar superpartners, right $\tilde{t}_R$ and left $\tilde{t}_L$, which become mixed as a result of the spontaneous breaking of $SU(2)\otimes U(1)$ symmetry, and this results in the formation of two charged scalar particles having masses $m_{\tilde{t}_1}$ and $m_{\tilde{t}_2}$:

$$m^2_{\tilde{t}_1,\tilde{t}_2}=\frac{1}{2}(m_Q^2+m_U^2+2m_t^2\pm\sqrt{(m_Q^2-m_U^2)^2+4m_t^2X_t^2})\,, \tag{13}$$

where $X_t = A_t + \mu_{\rm eff}/\tan\beta$. Since $m^2_{\tilde{t}_1}$ and $m^2_{\tilde{t}_2}$ should be positive we have $X_t^2<\dfrac{1}{m_t^2}(m_Q^2+m_t^2)(m_U^2+m_t^2)$. Otherwise, the quark fields acquire nonzero vacuum expectation values and the gauge $SU(2)\otimes U(1)$ symmetry of the initial Lagrangian is completely broken, which leads to the appearance of nonzero masses for gluons and photons. The contribution of the one–loop corrections from the t–quark and its superpartners to the effective interaction potential of the Higgs fields is expressed only in terms of their masses:

$$\begin{aligned}\Delta V(H_1,H_2,Y)=\frac{3}{32\pi^2}\left(m^4_{\tilde{t}_1}\left(\log\frac{m^2_{\tilde{t}_1}}{q^2}-\frac{3}{2}\right)+\right.\\ \left.+m^4_{\tilde{t}_2}\left(\log\frac{m^2_{\tilde{t}_2}}{q^2}-\frac{3}{2}\right)-2m_t^4\left(\log\frac{m_t^2}{q^2}-\frac{3}{2}\right)\right)\,.\end{aligned} \tag{14}$$

For this reason all Δ_i are merely functions of $\mu_{\rm eff}$ and do not depend on B_0 and y.

Using the first equation of the system (12) we can find $\mu_{\rm eff}$. In this case, the sign of $\mu_{\rm eff}$ is not fixed and must be considered as a free parameter in the theory. Substituting this value of $\mu_{\rm eff}$ into the two remaining equations of the system (12), we can eliminate B_0 from the number of independent fundamental parameters and calculate the vacuum expectation value of the field Y: $\langle Y\rangle = y/\sqrt{2}$ and in order to find B_0 we need to bear in mind the relationships linking the bilinear interaction constants B and B' in the electroweak scale with B_0 obtained by solving the renormalisation group equations in the modified NMSSM (see Appendix):

$$\begin{aligned}B(t)&=\zeta(t)B_0+\sigma(t)A+\omega(t)M_{1/2},\\ B'(t)&=B_0+\sigma_1(t)A\frac{\mu_0}{\mu_0'}+\omega_1(t)M_{1/2}\frac{\mu_0}{\mu_0'}\,,\end{aligned} \tag{15}$$

where $\zeta(t)$, $\sigma(t)$, $\sigma_1(t)$, $\omega(t)$, and $\omega_1(t)$ are various functions of t, $h_t(0)$, and $\lambda(0)$ which do not depend on the choice of fundamental soft supersymmetry breaking parameters on the grand unification scale, and also on μ_0 and μ_0'. For a fixed sign of $\mu_{\rm eff}$ three different solutions of the system of Eqs.(12) exist. However, only one of these is of interest from the physical point of view. It follows from the last equation in (12) that the vacuum expectation value of the field Y is of the order of $y\sim\lambda v^2/M_S$ and for the case of heavy supersymmetric particles $y\ll v$. The other two solutions give an excessively light CP–even Higgs boson which corresponds to fine tuning between the fundamental parameters B_0 and μ'.

The parameters $\mu_{\rm eff}$ and B_0 thus determined and also the vacuum expectation value y depend on the choice of A, m_0, and μ'. Thus, at the next stage of the analysis of the

modified NMSSM we studied the dependence of the particle spectrum on these fundamental parameters using Eqs.(6) linking $A_i(t_0)$ and $\mathfrak{M}_i^2(t_0)$ to A and m_0^2. Similarly we investigated the spectrum of superpartners of observable particles and Higgs bosons for other values of the Yukawa constants from the vicinity of the infrared quasi-fixed point. Although for $\tan\beta \leq 2$ in the strong Yukawa coupling regime the parameters h_t and λ can be selected so that the Yukawa constants of a b–quark and a τ–lepton would be the same on the grand unification scale [21], [22], when studying the particle spectrum in the modified NMSSM we do not confine ourselves to the case $R_{b\tau}(0) = 1$ where $R_{b\tau} = h_b(t)/h_\tau(t)$. This condition arises in minimal schemes of gauge interaction unification [23] and imposes very stringent constraints on the parameter space of the model being studied. However, since h_b and h_τ for $\tan\beta \sim 1$ have small absolute values, they can be generated by means of nonrenormalisable operators as a result of the spontaneous symmetry breaking on the scale M_X and in this case, the Yukawa constants of a b–quark and a τ–lepton may differ.

5 Calculations of masses of Higgs bosons and neutralinos

We shall first consider the Higgs sector in the modified NMSSM which includes three CP–even states, two CP–odd, and one charged Higgs boson. The determinants of the mass matrices of the CP–odd and charged Higgs bosons go to zero which corresponds to the appearance of two Goldstone bosons which are eaten by massive vector $W^\pm$ and Z bosons during spontaneous breaking of $SU(2) \otimes U(1)$ symmetry. The (3×3) mass matrix of the CP–odd sector is formed by mixing the imaginary parts of the neutral components of the Higgs doublets with the imaginary part of the field Y. However, since the determinant of this matrix is zero, the problem of finding the eigenvalues reduces to solving an ordinary quadratic equation. The calculated masses of the CP–odd states in the modified NMSSM are

$$m^2_{A_1,A_2} = \frac{1}{2}\left(m_A^2 + m_B^2 \pm \sqrt{\left(m_A^2 - m_B^2\right)^2 + 4\left(\frac{\lambda v}{\sqrt{2}}(\mu' + A_\lambda) + \Delta_0\right)^2}\right), \tag{16}$$

$$m_A^2 = m_1^2 + m_2^2 + 2\mu_{\rm eff}^2 + \frac{\lambda^2}{2}v^2 + \Delta_A, \quad m_B^2 = m_y^2 + \mu'^2 - B'\mu' + \frac{\lambda^2}{2}v^2 + \Delta_3,$$

where Δ_i are the one–loop corrections [33],[36],[38].

A more complex situation is encountered in the sector of the CP–even Higgs fields which appears as a result of mixing of the real parts of the neutral components of the Higgs doublets and the field Y. The determinant of the mass matrix of the CP–even sector ix nonzero and thus in order to calculate its eigenvalues we need to solve a cubic equation. However, for the case of heavy supersymmetric particles ($M_S \gg M_Z$) in the Higgs field

basis

$$\begin{aligned}
\chi_1 &= \frac{1}{\sqrt{2}}\cos\beta\,\mathrm{Re}\,H_1^0 + \frac{1}{\sqrt{2}}\sin\beta\,\mathrm{Re}\,H_2^0 \\
\chi_2 &= -\frac{1}{\sqrt{2}}\sin\beta\,\mathrm{Re}\,H_1^0 + \frac{1}{\sqrt{2}}\cos\beta\,\mathrm{Re}\,H_2^0 \\
\chi_3 &= \frac{1}{\sqrt{2}}\mathrm{Re}\,Y
\end{aligned} \tag{17}$$

this matrix has a hierarchical structure:

$$M_{ij}^2 = \begin{pmatrix} E_1^2 & 0 & 0 \\ 0 & E_2^2 & 0 \\ 0 & 0 & E_3^2 \end{pmatrix} + \begin{pmatrix} V_{11} & V_{12} & V_{13} \\ V_{21} & V_{22} & V_{23} \\ V_{31} & V_{32} & V_{33} \end{pmatrix} \tag{18}$$

where

$$\begin{gathered}
E_1^2 = 0, \qquad E_2^2 = m_1^2 + m_2^2 + 2\mu_{\mathrm{eff}}^2, \qquad E_3^2 = m_y^2 + \mu'^2 + B'\mu' \,, \\
V_{11} = M_Z^2\cos^2 2\beta + \frac{1}{2}\lambda^2 v^2 \sin^2 2\beta + \Delta_{11}, \qquad V_{13} = V_{31} = \lambda v X_1 + \Delta_{13} \,, \\
V_{12} = V_{21} = \left(\frac{1}{4}\lambda^2 v^2 - \frac{1}{2}M_Z^2\right)\sin 4\beta + \Delta_{12}, \qquad V_{23} = V_{32} = \lambda v X_2 + \Delta_{23} \,, \\
V_{22} = M_Z^2 \sin^2 2\beta + \frac{1}{2}\lambda^2 v^2 \cos^2 2\beta + \Delta_A + \Delta_{22}, \qquad V_{33} = \frac{1}{2}\lambda^2 v^2 + \Delta_{33} \,.
\end{gathered}$$

Here Δ_A and Δ_{ij} are the corrections from loops containing the t–quark and its superpartners. The hierarchical structure of the mass matrix means that perturbation from quantum mechanics can be used to diagonalise it. The role of the smallness parameters in the perturbation theory are played by the ratios M_Z^2/E_2^2 and M_Z^2/E_3^2. This method of calculating the masses of Higgs bosons in supersymmetric theories was developed in [24]. Also discussed there is the simplest method of obtaining a hierarchical mass matrix in the Higgs field basis (17). A numerical analysis made in this study showed that perturbation theory can be used to calculate the masses of Higgs bosons in the modified NMSSM to within 1 GeV ($\sim 1\%$).

This method can be used to diagonalise the mass matrix of a neutralino which occurs as a result of the mixing of superpartners of gauge bosons W_3 and B (or Z and γ) with superpartners of neutral Higgs fields. In the basis $(\tilde{B}^0, \tilde{W}^3, \tilde{H}_1^0, \tilde{H}_2^0, \tilde{Y})$, this mass matrix has the following form:

$$\tilde{M}_{ij} = \begin{pmatrix}
M_1 & 0 & -A' & B' & 0 \\
0 & M_2 & C' & -D' & 0 \\
-A' & C' & 0 & \mu_{\mathrm{eff}} & \frac{\lambda}{\sqrt{2}}v\sin\beta \\
B' & -D' & \mu_{\mathrm{eff}} & 0 & \frac{\lambda}{\sqrt{2}}v\cos\beta \\
0 & 0 & \frac{\lambda}{\sqrt{2}}v\sin\beta & \frac{\lambda}{\sqrt{2}}v\cos\beta & \mu'
\end{pmatrix} \tag{19}$$

The fragment of the (4×4) matrix which includes the first four columns and for rows is the same as the mass matrix of a neutralino in the MSSM with A', B', C', and D' given

by

$$A' = M_Z \cos\beta \sin\theta_W, \qquad B' = M_Z \sin\beta \sin\theta_W,$$
$$C' = M_Z \cos\beta \cos\theta_W, \qquad D' = M_Z \sin\beta \cos\theta_W.$$

Using the unitary transformation U:

$$U = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

the matrix (19) can be reduced to the form (18) $\tilde{M}' = U\tilde{M}U^{+}$ and then using the ratios $(M_Z/\mu_{\rm eff})^2$ and $(M_Z/\mu')^2$ as the small parameters in the first order of perturbation theory for the spectrum of supersymmetric particles we obtain

$$\begin{aligned}
m_{\tilde{\chi}_1} &= M_1 + \frac{(A'-B')^2}{2(M_1-\mu_{\rm eff})} + \frac{(A'+B')^2}{2(M_1+\mu_{\rm eff})}\,, \\
m_{\tilde{\chi}_2} &= M_2 + \frac{(C'-D')^2}{2(M_2-\mu_{\rm eff})} + \frac{(C'+D')^2}{2(M_2+\mu_{\rm eff})}\,, \\
m_{\tilde{\chi}_3} &= \mu_{\rm eff} + \frac{(A'-B')^2}{2(\mu_{\rm eff}-M_1)} + \frac{(C'-D')^2}{2(\mu_{\rm eff}-M_2)} + \frac{\lambda^2 v^2 \sin^2(\beta+\pi/4)}{2(\mu_{\rm eff}-\mu')} \\
m_{\tilde{\chi}_4} &= -\mu_{\rm eff} - \frac{(A'+B')^2}{2(M_1+\mu_{\rm eff})} - \frac{(C'+D')^2}{2(M_2+\mu_{\rm eff})} - \frac{\lambda^2 v^2 \sin^2(\beta-\pi/4)}{2(\mu_{\rm eff}+\mu')}, \\
m_{\tilde{\chi}_5} &= \mu' + \frac{\lambda^2 v^2 \sin^2(\beta+\pi/4)}{2(\mu'-\mu_{\rm eff})} + \frac{\lambda^2 v^2 \sin^2(\beta-\pi/4)}{2(\mu'+\mu_{\rm eff})}\,.
\end{aligned} \tag{20}$$

The accuracy with which $m_{\tilde{\chi}_i}$ is calculated is slightly lower than that for the CP–even Higgs sector. This primarily because the parameters used for the expansion when diagonalising the neutralino mass matrix according to perturbation theory are larger. At this point we shall not discuss the spectrum of squarks, sleptons, and charginos in greater detail since the analytic expressions for the masses of these particles remain the same as in MSSM. In this case, in all the formulas we need to replace μ with $\mu_{\rm eff}$. We merely note that in the principal approximation the masses of two Dirac charginos and neutralinos $\tilde{\chi}_2$ and $\tilde{\chi}_3$ are the same: $m_{\tilde{\chi}^{\pm}_1} \approx \mu_{\rm eff}$ and $m_{\tilde{\chi}^{\pm}_2} \approx M_2$.

6 Results of the numerical analysis

Results of a numerical analysis of the spectrum of Higgs bosons and superpartners of observable particles in the modified NMSSM are given in Figs.1–5 and Tables 1–3. We first need to note that for a fixed sign of $\mu_{\rm eff}$ there are two allowed regions of parameter space. In one of these the mass of the lightest Higgs boson is greater than in the MSSM (see Figs. 1a and 3a) whereas in the other it is smaller (see Figs. 1b and 3b). The mass

of the lightest calculated in the first order with respect to perturbation theory has the following form:

$$m_h^2 \approx V_{11} - \frac{|V_{13}|^2}{E_3^2} = M_Z^2 \cos^2 2\beta + \frac{1}{2}\lambda^2 v^2 \sin^2 2\beta + \Delta_{11} - \frac{(\lambda v X_1 + \Delta_{13})^2}{m_y^2 + \mu'^2 + B'\mu'}\,. \qquad (21)$$

Since the matrix element $V_{12} \sim M_Z^2$, we neglected its contribution to m_h. The mass of the lightest CP–even Higgs boson reaches its highest value when $\mu' = -\dfrac{2\mu_{\rm eff}}{\sin 2\beta} - A_\lambda - \dfrac{\sqrt{2}\Delta_{13}}{\lambda v \sin 2\beta}$, when $V_{13} = 0$ (see Figs. 1a and 3a). Thus, m_h is larger in that region of parameter space where the signs of μ' and $\mu_{\rm eff}$ are opposed. In the limit $\mu' \to \pm\infty$ the masses of the CP–even and CP–odd Higgs bosons corresponding to the field Y become much larger than the scale of the supersymmetry breaking. In the low energy region their contribution to the effective interaction potential of the Higgs fields H_1 and H_2 disappears and the mass of the lightest Higgs boson is the same as in the MSSM. For this reason, as can be seen from the graphs plotted in Figs. 1a,1b and 3a,3b, m_h reaches a constant value when $\mu' \to \pm\infty$. The results of the numerical calculations plotted in these figures indicate that the two–loop corrections [11] play a significant role in the calculations of the mass of the lightest Higgs boson. In this particular case, they reduce its mass approximately by 10 GeV. Although the one–loop corrections increase logarithmically as the scale of the supersymmetry breaking increases, their increase for $M_S \gg M_Z$ is completely compensated by the log–log asymptotic form of the two–loop corrections and m_h remains almost constant. Allowance for the loop corrections has the result that for $\mu_{\rm eff} < 0$ the mass of the lightest Higgs boson is greater than that in the case $\mu_{\rm eff} > 0$. This can be attributed to the fact that m_h increases with increasing mixing in the superpartner sector of the t–quark ($\tilde{t}_R$ and $\tilde{t}_L$) which is determined by the value of $X_t = A_t + \mu_{\rm eff}/\tan\beta$. Since $A_t < 0$, the absolute value of the mixing between $\tilde{t}_R$ and $\tilde{t}_L$ is greater for $\mu_{\rm eff} < 0$. It should be noted that the mass of the lightest Higgs boson is almost independent of A and m_0 because of the weak dependence of the squark mass on the corresponding fundamental parameters (see Tables 1 and 2).

Since that part of the parameter space in which $\mu_{\rm eff}$ and μ' have the same sign is almost eliminated by the existing experimental data, it is most interesting to study the spectrum of Higgs bosons in the region where the mass of the lightest Higgs boson is greater than that in the minimal supersymmetric model. In this particular region of parameter space the bilinear soft supersymmetry breaking constants and $\mu_{\rm eff}$ have opposite signs and near the maximum of m_h the parameter is $\mu' \sim -2\mu_{\rm eff}/\sin 2\beta$ ($|\mu'| > |\mu_{\rm eff}| \sim M_S$). For this reason the heaviest particle in the modified NMSSM spectrum is the CP–even Higgs boson which corresponds to the neutral field Y sice its mass in the principal approximation with respect to perturbation theory is $M_S^2 \approx E_3^2 > \mu'^2$ and is substantially larger than the scale of the supersymmetry breaking. It can be seen from Figs. 2a and 4a that the mass of the other heavy CP–even Higgs boson (m_H) is almost independent of μ' since $m_S^2 \gg m_H^2$. However, the spectrum of the CP–odd Higgs sector is determined to a considerable by the choice of fundamental parameters. As μ' increases, the mass of the CP–odd Y increases and the latter becomes one of the heaviest particles. For values of $\mu' \sim B'$ the mass of the lightest CP–odd Higgs boson $m_{A_2}^2 \approx m_B^2$ is very low (see Figs. 2a, 4a), which leads to the appearance of a constraint on μ'. Nevertheless, with this choice of fundamental parameters this Higgs boson is negligibly involved in electromagnetic and weak interactions since the main contribution to its wave function is made by the CP–odd component of the field Y. Thus, even when its mass is relatively low, it is extremely

difficult to detect this particle experimentally. the heaviest fermion in this model is the superpartner of the field Y. Its mass $m_{\tilde{\chi}_5}$ is proportional to μ' (see (20)). The spectrum of remaining neutralinos, charginos, squarks, and sleptons does not depend on the choice of μ'.

Since the dependence of the soft supersymmetry breaking parameters on A disappears in the strong Yukawa coupling regime on the electroweak scale, the spectrum of superpartners of the observable particles and also μ_{eff} and B whose numerical values are determined by solving the system of Eqs.(12), vary weakly when the trilinear interaction constant of the scalar fields varies between $-M_{1/2}$ and $M_{1/2}$. Despite this, the dependence of the Higgs boson spectrum on A is conserved. This is mainly because the bilinear interaction constant of the neutral scalar fields B' is proportional to A. Using the relations (15), we obtain

$$B'(t_0) = \frac{1}{\zeta(t_0)}B(t_0) + \left[\left(\sigma_1(t_0)\frac{\mu_0}{\mu_0'} - \frac{\sigma(t_0)}{\zeta(t_0)}\right)x + \left(\omega_1(t_0)\frac{\mu_0}{\mu_0'} - \frac{\omega(t_0)}{\zeta(t_0)}\right)\right]M_{1/2},$$

where $x = A/M_{1/2}$. As x increases for $\mu_{\text{eff}} < 0$ ($\mu_{\text{eff}} > 0$) the bilinear interaction constant decreases (increases) in absolute value and conversely. As $|B'|$ decreases, the masses of the CP–even and CP–odd states corresponding to the neutral field Y converge. At the same time, an increase in the absolute value of B' leads to a decrease in $m^2_{A_2}$ which disappears when $B' \sim \mu'$. The dependence of the Higgs boson spectrum of the parameter A for $m_0 = 0$ is studied in Figs. 2b and 4b. The parameter μ' in this particular case is selected so that the mass of the lightest Higgs boson coincides with the upper bound on m_h for $A = 0$.

Although in some cases we assumed $m_0 = 0$ when analysing the modified NMSSM, this limit is unacceptable from the physical point of view since in this case the lightest (and consequently stable) supersymmetric particle is the superpartner of the right τ–lepton which contradicts existing astrophysical observations. However, as m_0 increases the mass of the superpartner of the right τ–lepton increases and even for comparatively low values of $m_0/M_{1/2}$ the lightest particle in the spectrum of superpartners of observable particles becomes the neutralino. The results of the numerical calculations presented in Tables 1 and 2 can be used to assess the influence of the fundamental constants A, m_0, and $M_{1/2}$ on the superpartner spectrum of the t–quark, gluinos, neutralinos, charginos, and Higgs bosons. For each set of parameters listed above we give the values of the upper bound on the mass of the lightest Higgs boson. calculated in the one–loop and two–loop approximations and also the corresponding μ_{eff}, B_0, y, and μ' for which $V_{13} = 0$. It can be seen from the data presented in Table 1 that the qualitative pattern of the spectrum remains unchanged if the parameters A and m_0 vary within reasonable limits. It should also be noted that as m_0^2 increases, the masses of squarks, sleptons, Higgs bosons, and also heavy charginos and neutralinos increases whereas the spectrum of the lightest particles remains unchanged. The mass of a charged Higgs boson which has not been mentioned before is almost independent of A and μ' and numerically similar results are obtained for the mass of a charged Higgs boson $m_{H^\pm}$.

In the present paper we have made a detailed study of the superpartner and Higgs boson spectrum for initial values of the Yukawa constants $h_t^2(0) = \lambda^2(0) = 10$ corresponding to the scenario of the infrared quasi-fixed point in the NMSSM. The results of the numerical calculations presented in Tables 1 and 2 indicate that for $m_t(M_t^{\text{pole}})$ and $m_3 \leq 2$ TeV the mass of the lightest Higgs boson does not exceed 127 GeV. Other data presented in Table

3 indicate that the distinguishing features of the supersymmetric particle spectrum are conserved for $h_t^2(0) \gg \lambda^2(0)$ and $h_t^2(0) \ll \lambda^2(0)$ as along as the Yukawa constants on the grand unification scale are substantially larger than the gauge constants. Nevertheless, the upper bound on the mass of the lightest Higgs boson, the value of $\tan\beta$, and the particle masses calculated for $\mu' = -\frac{2\mu_{\rm eff}}{\sin 2\beta} - A_\lambda - \frac{\sqrt{2}\Delta_{13}}{\lambda v \sin 2\beta}$, when $V_{13} = 0$ vary as a function of the choice of $h_t^2(0)$ and $\lambda^2(0)$. Nevertheless, as $\lambda^2(0)$ decreases from 10 to 2, the upper bound on m_h for $M_3 = 1$ TeV drops from 128 to 113 GeV (see Table 3). Thus, at the concluding stage of the analysis of the modified NMSSM for each fixed $\tan\beta$ we selected the Yukawa constant $\lambda(t_0)$ so that m_h reached its highest value on condition that perturbation theory can be applied as far as the grand unification scale. The dependence $m_h(\tan\beta)$ thus obtained is plotted in Fig. 5 where we also plotted the upper bound m_h in the MSSM as a function of $\tan\beta$. As was to be expected, the two bounds on the mass of the lightest Higgs boson are almost the same for large $\tan\beta$ when the term $\frac{\lambda^2 v^2}{2}\sin^2 2\beta$ in Eq.(5) tends to zero. The curve $m_h(\tan\beta)$ in the NMSSM reaches its maximum when $\tan\beta \sim 2.5$ which corresponds to the strong Yukawa coupling regime. Both bounds on the mass of the lightest Higgs boson were obtained for $M_3 \leq 2$ TeV. By varying the scale of supersymmetry breaking we can show that the mass of the lightest Higgs boson in the NMSSM does not exceed 130.5 ± 3.5 GeV. The indeterminacy observed in calculations of the upper bound on m_h is mainly attributable to the experimental error with which the mass of a t–quark is measured.

7 Conclusions

In the nonminimal supersymmetric model the mass of the lightest Higgs boson reaches its highest value in the strong Yukawa coupling regime when all the solutions of the renormalisation group equations are grouped near the infrared quasi-fixed point. However, in this region of the parameter space using the NMSSM with a minimal set of fundamental parameters it is not possible to obtain a self-consistent solution which on the one hand would give a spectrum with heavy supersymmetric particles and on the other could give a mass of the lightest Higgs boson greater than that in the MSSM. In order to find such a solution, we need to modify the nonminimal supersymmetric model. In the present paper we studied the spectrum of superpartners and Higgs bosons using a very simple expansion of he NMSSM which can give a self-consistent solution in the strong Yukawa coupling regime. Although the parameter space of this model is expanded substantially, the theory does not lose its predictive capacity.

The mass matrix of the CP–even Higgs sector in the modified NMSSM has a hierarchical structure which means that it can be diagonalised using a method of calculating the spectrum of Higgs bosons proposed earlier, which is based on the ordinary perturbation theory of quantum mechanics. This method can be used to calculate the mass of Higgs bosons to within 1 GeV ($\sim 1\%$). In this case the mass of the lightest Higgs boson near the infrared quasi-fixed point for $m_t(M_t^{\rm pole}) = 165$ GeV and $M_3 \leq 2$ TeV does not exceed 127 GeV. By varying the ratio of the Yukawa constants on the grand unification scale, we can show that $m_h \leq 130.5 \pm 3.5$ GeV where the indeterminacy observed when calculating the upper bound on m_h is mainly attributable to the experimental error with which the

mass of the t–quark is measured. The heaviest particle in the region of the parameter space of interest is the CP–even Higgs boson corresponding to the neutral field Y.

In the present study we used the same method of diagonalising the mass matrices to calculate and study neutralino masses. As a result we showed that the heaviest fermion in the dominant region of parameter space is $\tilde{Y}$, the superpartner of the neutral scalar field Y. For values of $m_0^2 \le M_{1/2}^2$ gluinos, squarks, heavy CP–even and CP–odd Higgs bosons are substantially heavier than sleptons, lightest charginos, and neutralinos. The only exception is one of the CP–odd Higgs bosons whose mass varies substantially depending on the choice of parameters of the model. However, even if it is relatively low, for example, of the order of M_Z, there are certain problems involved in recording it experimentally since the main contribution to its wave function is made by the CP–odd component of the field Y.

The upper bound on the mass of the lightest Higgs boson in the nonminimal supersymmetric model was also studied on recent publications [25] and [62]. The predictions obtained in these studies are $5-6$ GeV higher than the bound given above. The difference in the predictions can be attributed to the fact that the authors of [25] and [62] used the value of $|X_t/M_S| = \sqrt{6}$ where $M_S = \sqrt{m_{\tilde{t}_1} m_{\tilde{t}_2}}$ to calculate the upper bound on m_h since the mass of the lightest Higgs boson reaches its highest value for this value of X_t. However, in the strong Yukawa coupling regime in the modified NMSSM the ratio $|X_t/M_S|$ is $1.4-1.5$. Since the mass of the lightest Higgs boson increases with increasing mixing between the t–quark superpartners for $0 \le |X_t/M_S| \le \sqrt{6}$, and the ratio $|X_t/M_S|$ is considerably less than $\sqrt{6}$, the upper bound on m_h in the realistic expansion of the NMSSM is more stringent that the absolute bound in the nonminimal supersymmetric model.

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Appendix

Renormalisation group equations for μ, μ', B, and B' parameters in the modified NMSSM and their solutions.

Side by side with the trilinear couplings and masses of scalar particles the modified NMSSM, within which we investigate the particle spectrum, contains the parameters μ, μ', B, and B'. An evolution of these constants is described by four renormalisation group equations:

$$\begin{aligned}
\frac{d\mu}{dt} &= -\frac{\mu}{2}\left(2Y_\lambda + 3Y_t - 3\tilde{\alpha}_2 - \frac{3}{5}\tilde{\alpha}_1\right), \\
\frac{d\mu'}{dt} &= -2\mu'(Y_\lambda + Y_\varkappa), \\
\frac{dB}{dt} &= -\left(2Y_\lambda B + \sqrt{Y_\lambda Y_\varkappa}B'\frac{\mu'}{\mu} + 3Y_t A_t + 2Y_\lambda A_\lambda - 3\tilde{\alpha}_2 M_2 - \frac{3}{5}\tilde{\alpha}_1 M_1\right), \\
\frac{dB'}{dt} &= -\left(2Y_\varkappa B' + 4\sqrt{Y_\lambda Y_\varkappa}B\frac{\mu}{\mu'} + 4Y_\lambda A_\lambda\frac{\mu}{\mu'} + 4Y_\varkappa A_\varkappa\right).
\end{aligned} \tag{22}$$

For $\varkappa = 0$ with the minimal set of fundamental parameters $B(0) = B'(0) = B_0$, $A_i(0) = A$, $M_i(0) = M_{1/2}$, $\mu'(0) = \mu'_0$, and $\mu(0) = \mu_0$, one can show using a general solution of the system of linear differential equations, that

$$\begin{aligned}
\mu(t) &= \xi(t)\mu_0, \\
\mu'(t) &= \xi_1(t)\mu'_0, \\
B(t) &= \zeta(t)B_0 + \sigma(t)A + \omega(t)M_{1/2}, \\
B'(t) &= B_0 + \sigma_1(t)A\frac{\mu_0}{\mu'_0} + \omega_1(t)M_{1/2}\frac{\mu_0}{\mu'_0},
\end{aligned} \tag{23}$$

where the functions $\xi(t)$, $\xi_1(t)$, $\zeta(t)$, $\sigma(t)$, $\sigma_1(t)$, $\omega(t)$, and $\omega_1(t)$, which determine the evolution of the fundamental parameters, mainly depend on a choice of Yukawa constants on the grand unification scale and do not depend on the initial values of the soft SUSY breaking parameters, μ_0, and μ'_0.

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Table 1. Mass spectrum of superpartners of observable particles and Higgs bosons for $\lambda^2(0) = h_t^2(0) = 10$ and $\mu_{\rm eff} > 0$ depending upon the choice of fundamental parameters A, m_0, and $M_{1/2}$ (all parameters and masses are given in GeV).

m_0^2	0	$M_{1/2}^2$	0	0	0	0
A	0	0	$-M_{1/2}$	$0.5M_{1/2}$	0	0
$M_{1/2}$	-392.8	-392.8	-392.8	-392.8	-785.5	-196.4
$m_t(t_0)$	165	165	165	165	165	165
$\tan\beta$	1.883	1.883	1.883	1.883	1.883	1.883
$\mu_{\rm eff}$	728.6	841.7	726.8	730.1	1361.2	380.4
B_0	-1629.1	-1935.4	-1260.0	-1813.2	-3064.4	-861.8
y	-0.00037	-0.00021	-0.00043	-0.00035	-0.00006	-0.00233
$\mu'(t_0)$	-1899.8	-2176.7	-1905.9	-1898.3	-3544.6	-993.1
$\mathbf{m_h(t_0)}$ **(1–loop)**	**125.0**	**125.1**	**125.0**	**125.0**	**134.9**	**114.8**
$\mathbf{m_h(t_0)}$ **(2–loop)**	**118.4**	**118.5**	**118.4**	**118.4**	**123.2**	**111.9**
$M_3(1\ \mathrm{TeV})$	1000	1000	1000	1000	2000	500
$m_{\tilde{t}_1}(1\ \mathrm{TeV})$	840.6	889.7	841.1	840.3	1652.0	447.4
$m_{\tilde{t}_2}(1\ \mathrm{TeV})$	695.1	713.6	696.6	694.3	1366.2	371.6
$m_H(1\ \mathrm{TeV})$	898.5	1080.5	895.4	900.3	1691.0	468.8
$m_S(1\ \mathrm{TeV})$	2623.4	3034.3	2452.2	2706.0	4901.7	1378.0
$m_{A_1}(1\ \mathrm{TeV})$	953.9	1113.8	1245.7	925.2	1722.6	538.2
$m_{A_2}(1\ \mathrm{TeV})$	704.3	762.7	872.0	318.2	1366.2	302.2
$m_{\tilde{\chi}_1}(t_0)$	164.6	164.4	164.6	164.6	326.9	84.3
$m_{\tilde{\chi}_2}(t_0)$	327.8	327.6	327.8	327.8	649.4	170.1
$m_{\tilde{\chi}_3}(1\ \mathrm{TeV})$	755.1	870.8	753.3	756.7	1404.2	400.9
$\lvert m_{\tilde{\chi}_4}(1\ \mathrm{TeV})\rvert$	755.9	872.6	755.1	758.4	1405.0	404.3
$\lvert m_{\tilde{\chi}_5}(1\ \mathrm{TeV})\rvert$	1931.8	2212.3	1938	1930.3	3599.0	1015.4
$m_{\tilde{\chi}_1^{\pm}}(t_0)$	327.8	327.6	327.8	327.8	649.4	169.9
$m_{\tilde{\chi}_2^{\pm}}(1\ \mathrm{TeV})$	757.0	872.6	755.2	758.5	1405.2	404.5

Table 2. Mass spectrum of superpartners of observable particles and Higgs bosons for $\lambda^2(0) = h_t^2(0) = 10$ and $\mu_{\rm eff} < 0$ depending upon the choice of fundamental parameters A, m_0, and $M_{1/2}$ (all parameters and masses are given in GeV).

m_0^2	0	$M_{1/2}^2$	0	0	0	0
A	0	0	$-M_{1/2}$	$M_{1/2}$	0	0
$M_{1/2}$	-392.8	-392.8	-392.8	-392.8	-785.5	-196.4
$m_t(t_0)$	165	165	165	165	165	165
$\tan\beta$	1.883	1.883	1.883	1.883	1.883	1.883
$\mu_{\rm eff}$	-727.8	-840.9	-726.0	-731.2	-1360.7	-378.9
B_0	1008	1320.3	1366.7	647.9	2050.4	495.8
y	-0.00149	-0.001	-0.00128	-0.00177	-0.00020	-0.0112
$\mu'(t_0)$	1671.5	1950.6	1656.8	1690.3	3172.7	857.8
$\mathbf{m_h(t_0)}$ **(1–loop)**	**134.1**	**134.9**	**134.0**	**134.2**	**143.1**	**124.1**
$\mathbf{m_h(t_0)}$ **(2–loop)**	**124.4**	**124.8**	**124.3**	**124.5**	**127.2**	**119.6**
M_3(1 TeV)	1000	1000	1000	1000	2000	500
$m_{\tilde{t}_1}$(1 TeV)	890.2	935.6	890.5	889.8	1682.8	507.9
$m_{\tilde{t}_2}$(1 TeV)	630.3	652.2	632.2	628.0	1328.1	283.5
m_H(1 TeV)	896.2	1078.5	893.5	899.3	1689.9	464.4
m_S(1 TeV)	2147.4	2565.9	2309.2	1972.3	4126.5	1097.7
m_{A_1}(1 TeV)	1123.2	1219.3	931.0	1437.9	1984.8	623.1
m_{A_2}(1 TeV)	857.6	1017.8	545.0	886.9	1657.5	412.8
$m_{\tilde{\chi}_1}(t_0)$	160.0	160.5	160.0	160.0	324.4	74.9
$m_{\tilde{\chi}_2}(t_0)$	311.1	313.7	311.0	311.2	639.9	141.4
$\lvert m_{\tilde{\chi}_3}$(1 TeV)$\rvert$	753.7	896.6	751.9	757.2	1403.4	398.5
$m_{\tilde{\chi}_4}$(1 TeV)	764.7	878.1	763.0	768.1	1410.0	416.7
$m_{\tilde{\chi}_5}$(1 TeV)	1700.7	1983.2	1685.8	1719.6	3221.8	879.1
$m_{\tilde{\chi}_1^\pm}(t_0)$	310.7	313.4	310.7	310.8	639.8	139.4
$m_{\tilde{\chi}_2^\pm}$(1 TeV)	763.3	877.0	761.6	766.7	1409.1	414.5

Table 3. Mass spectrum of superpartners of observable particles and Higgs bosons for $A = m_0 = 0$, but for different initial values $h_t^2(0)$,$\lambda^2(0)$ (all parameters and masses are given in GeV).

	$\mu_{\rm eff} < 0$				$\mu_{\rm eff} > 0$			
$\lambda^2(0)$	0	2	10	10	0	2	10	10
$h_t^2(0)$	10	10	10	2	10	10	10	2
$M_{1/2}$	-392.8	-392.8	-392.8	-392.8	-392.8	-392.8	-392.8	-392.8
$\tan\beta$	1.614	1.736	1.883	2.439	1.614	1.736	1.883	2.439
$\mu_{\rm eff}$	-821.5	-771.4	-727.8	-641.8	822.7	772.4	728.6	642.3
B_0	471.7	622.5	1008.0	886.2	-743.1	-988.1	-1629.1	-1583.3
y	—	-0.0014	-0.0015	-0.0012	—	-0.0003	-0.0004	-0.0005
$\mu'(t_0)$	—	1693.9	1671.5	1749.8	—	-1941.4	-1899.8	-1943.1
$\mathbf{m_h(t_0)}$ **(1–loop)**	**103.5**	**123.6**	**134.1**	**137.6**	**88.1**	**112.4**	**125.0**	**131.2**
$\mathbf{m_h(t_0)}$ **(2–loop)**	**90.3**	**113.0**	**124.4**	**127.8**	**79.7**	**105.5**	**118.4**	**123.6**
$M_3(1\text{ TeV})$	1000	1000	1000	1000	1000	1000	1000	1000
$m_{\tilde{t}_1}(1\text{ TeV})$	894.0	891.6	890.2	890.5	834.6	837.0	840.6	853.5
$m_{\tilde{t}_2}(1\text{ TeV})$	613.5	622.2	630.3	648.5	692.2	693.8	695.1	696.4
$m_H(1\text{ TeV})$	1033.4	961.0	896.2	758.5	1035.7	963.3	898.5	761.1
$m_S(1\text{ TeV})$	—	1999.8	2147.4	2187.2	—	2405.3	2623.4	2663.8
$m_{A_1}(1\text{ TeV})$	1029.7	1374.8	1123.2	1294.0	1031.3	1390.6	953.9	965.1
$m_{A_2}(1\text{ TeV})$	—	949.8	857.6	735.6	—	951.6	704.3	674.3
$m_{\tilde{\chi}_1}(t_0)$	160.3	160.1	160.0	159.9	164.6	164.6	164.6	164.4
$m_{\tilde{\chi}_2}(t_0)$	312.7	311.9	311.1	309.4	328.2	328.1	327.8	326.4
$\lvert m_{\tilde{\chi}_3}(1\text{ TeV})\rvert$	842.8	795.8	753.7	665.8	844.4	797.2	755.1	668.1
$\lvert m_{\tilde{\chi}_4}(1\text{ TeV})\rvert$	856.4	807.8	764.7	677.1	850.6	800.9	755.9	666.7
$\lvert m_{\tilde{\chi}_5}(1\text{ TeV})\rvert$	—	1711.2	1700.7	1790.0	—	1960.7	1931.8	1986.5
$m_{\tilde{\chi}_1^\pm}(t_0)$	312.4	311.6	310.7	309.0	328.2	328.1	327.8	326.4
$m_{\tilde{\chi}_2^\pm}(1\text{ TeV})$	854.2	806.0	763.3	676.7	849.5	800.4	757.0	669.0

Figure captions

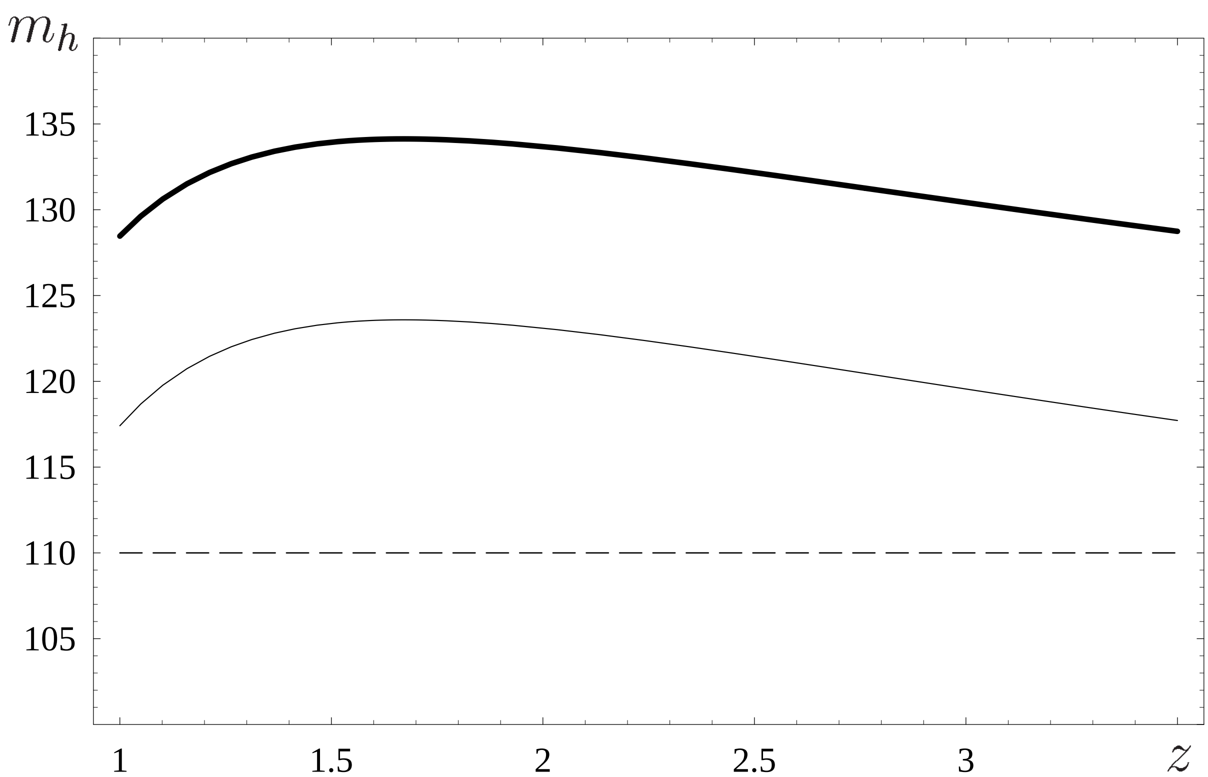
Fig.1. The lightest Higgs boson mass in modified NMSSM as a function of $z = \mu'/1$ TeV for $h_t^2(0) = \lambda^2(0) = 10$, $m_0 = 0$, $M_3 = 1$ TeV, and for $\mu_{\rm eff} \le 0$. Thick and thin curves meet calculations in one–loop and two–loop approximations, respectively.

Fig.2. Mass spectrum in modified NMSSM as a function of $z = \mu'/1$ TeV and $x = A/M_{1/2}$ for $h_t^2(0) = \lambda^2(0) = 10$, $m_0 = 0$, $M_3 = 1$ TeV, and for $\mu_{\rm eff} \le 0$. Thick and thin curves give the masses of heavy CP–even Higgs bosons. Dotted and dashed curves give the masses of CP–odd Higgs bosons. The dashdot curve gives the mass of the heaviest neutralino.

Fig.3. The lightest Higgs boson mass in modified NMSSM as a function of $z = \mu'/1$ TeV for $h_t^2(0) = \lambda^2(0) = 10$, $m_0 = 0$, $M_3 = 1$ TeV, and for $\mu_{\rm eff} \ge 0$. Thick and thin curves meet calculations in one–loop and two–loop approximations, respectively.

Fig.4. Mass spectrum in modified NMSSM as a function of $z = \mu'/1$ TeV and $x = A/M_{1/2}$ for $h_t^2(0) = \lambda^2(0) = 10$, $m_0 = 0$, $M_3 = 1$ TeV, and for $\mu_{\rm eff} \ge 0$. Thick and thin curves give the masses of heavy CP–even Higgs bosons. Dotted and dashed curves give the masses of CP–odd Higgs bosons. The dashdot curve gives the mass of the heaviest neutralino.

Fig.5. Upper bound on the mass of the lightest Higgs boson in the MSSM (thin curve) and in the modified NMSSM (thick curve) as a function of $\tan\beta$ for $M_3 = 2$ TeV.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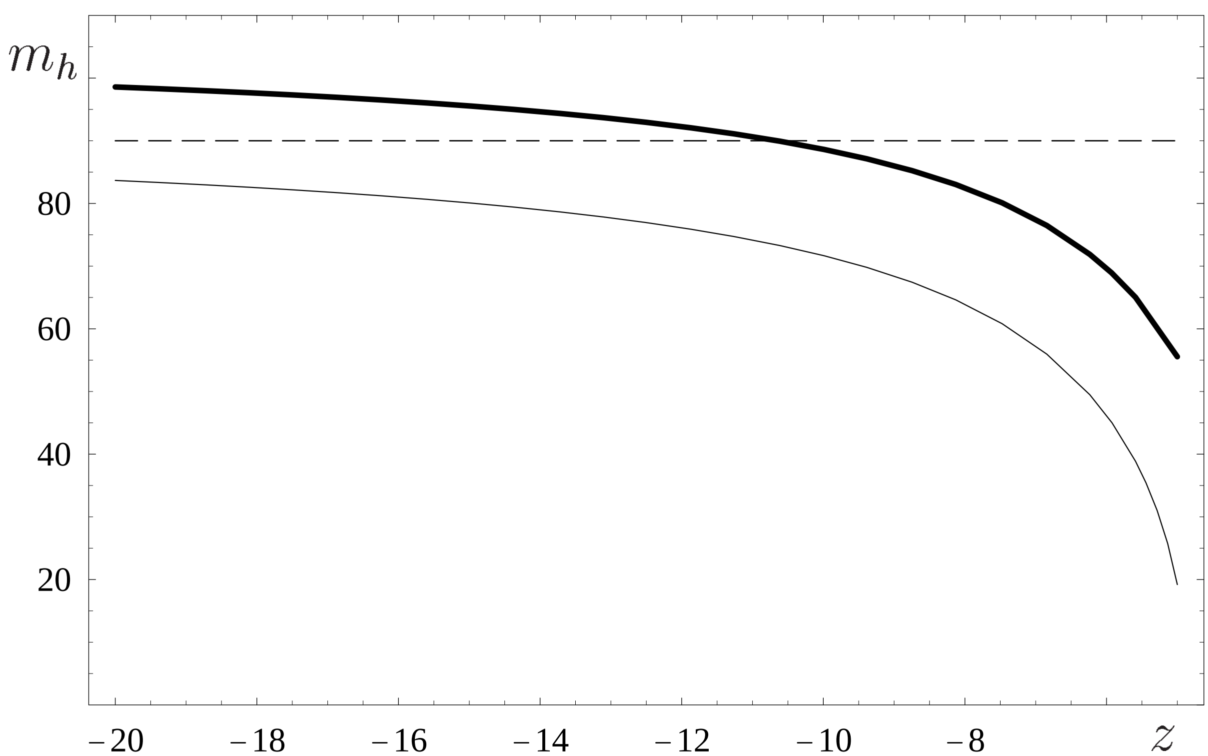
130
125
120
115
110
105
1
1.5
2
2.5
3
z

Fig.1a.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40
20
−20
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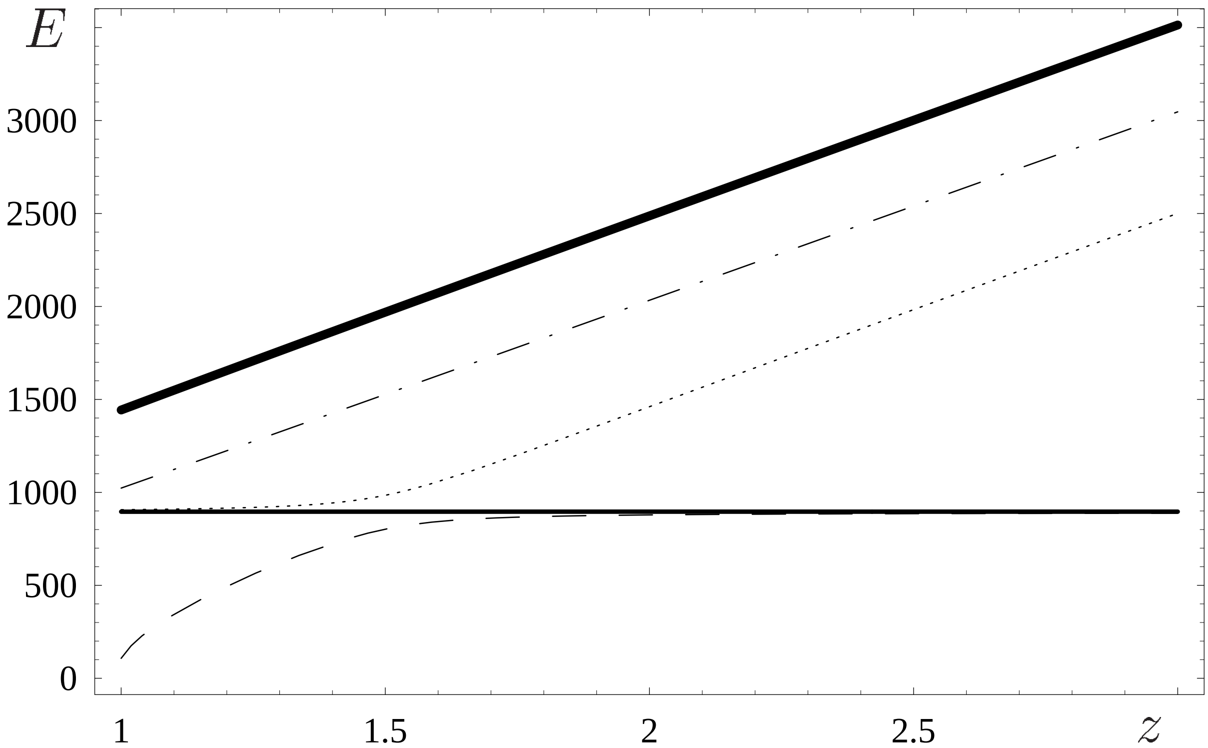
−16
−14
−12
−10
−8
z

Fig.1b.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2000
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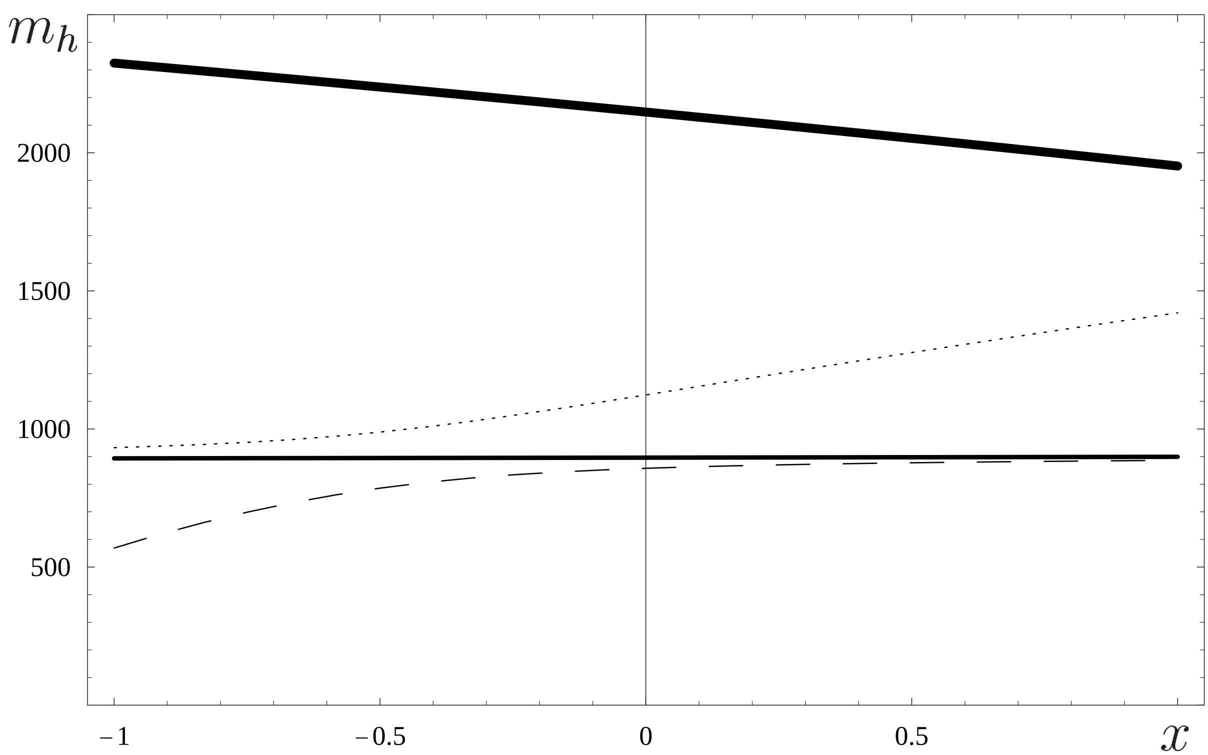
500
0
1
1.5
2
2.5
z

Fig.2a.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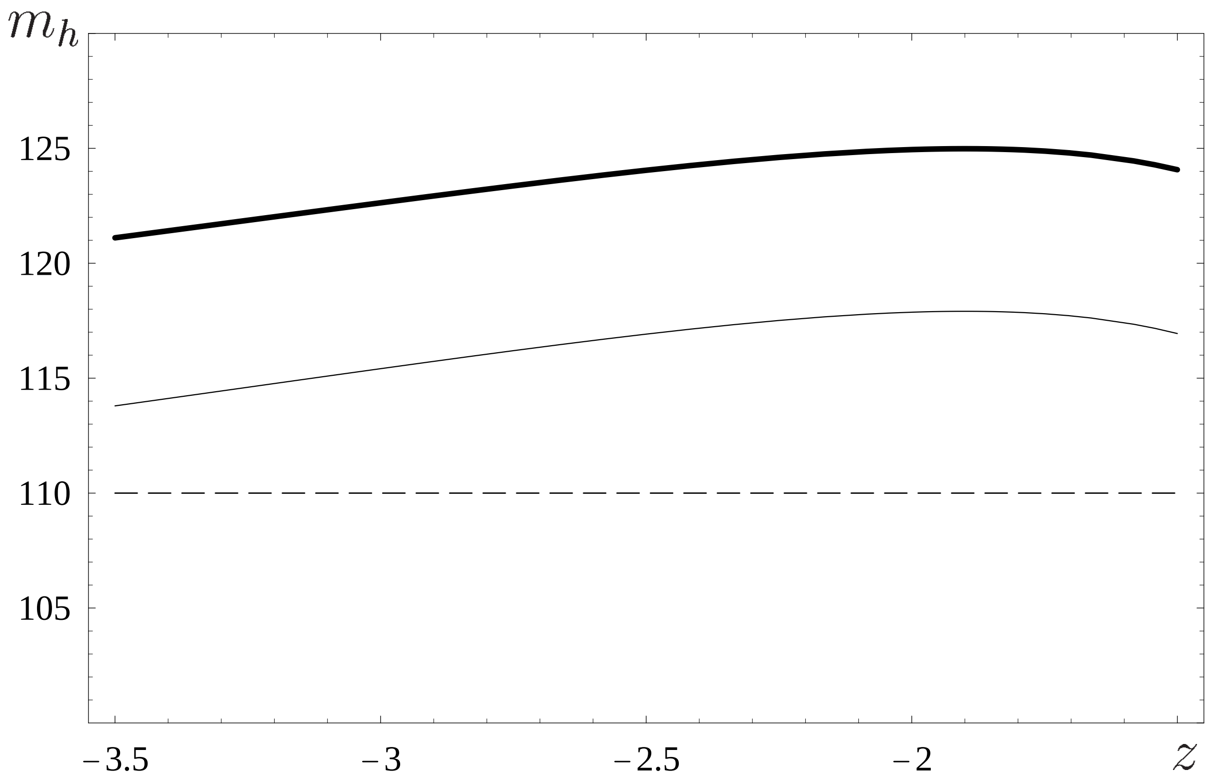
0
0.5
x

Fig.2b.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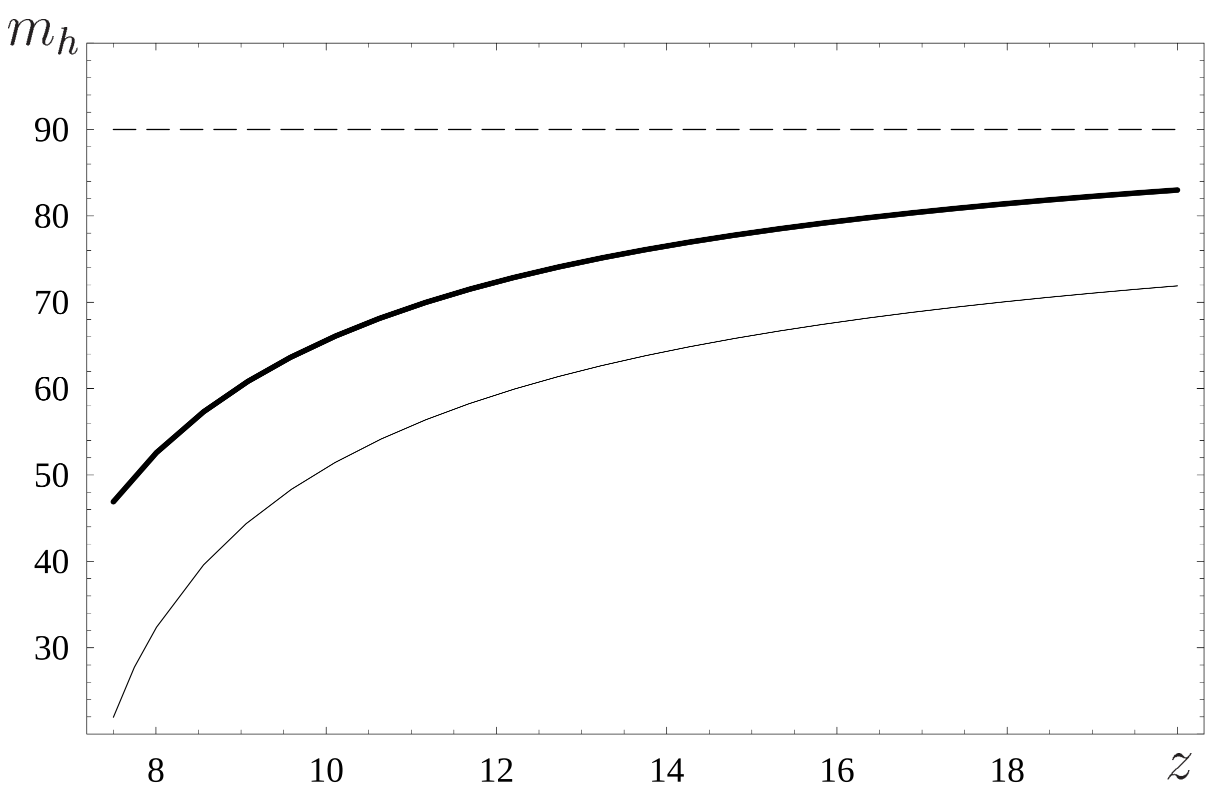
115
110
105
−3.5
−3
−2.5
−2
z

Fig.3a.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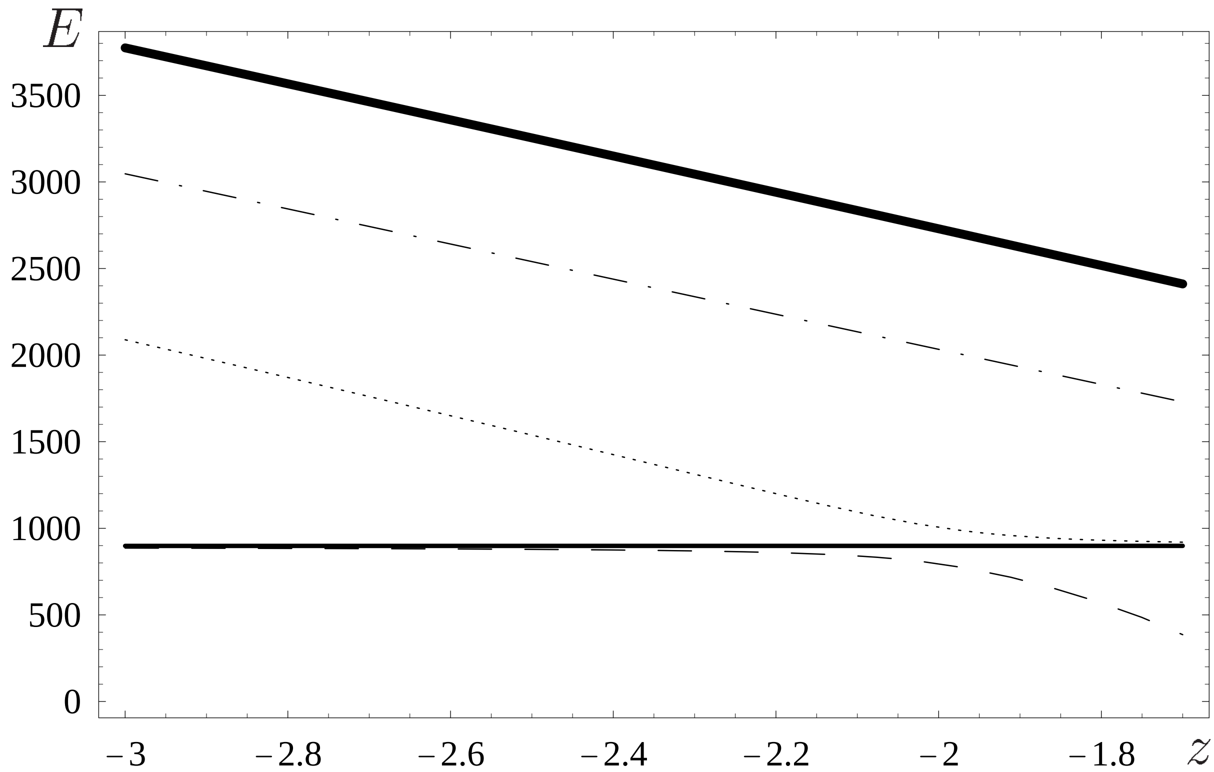
70
60
50
40
30
8
10
12
14
16
18
z

Fig.3b.



E
3500
3000
2500
2000
1500
1000
500
0
−3
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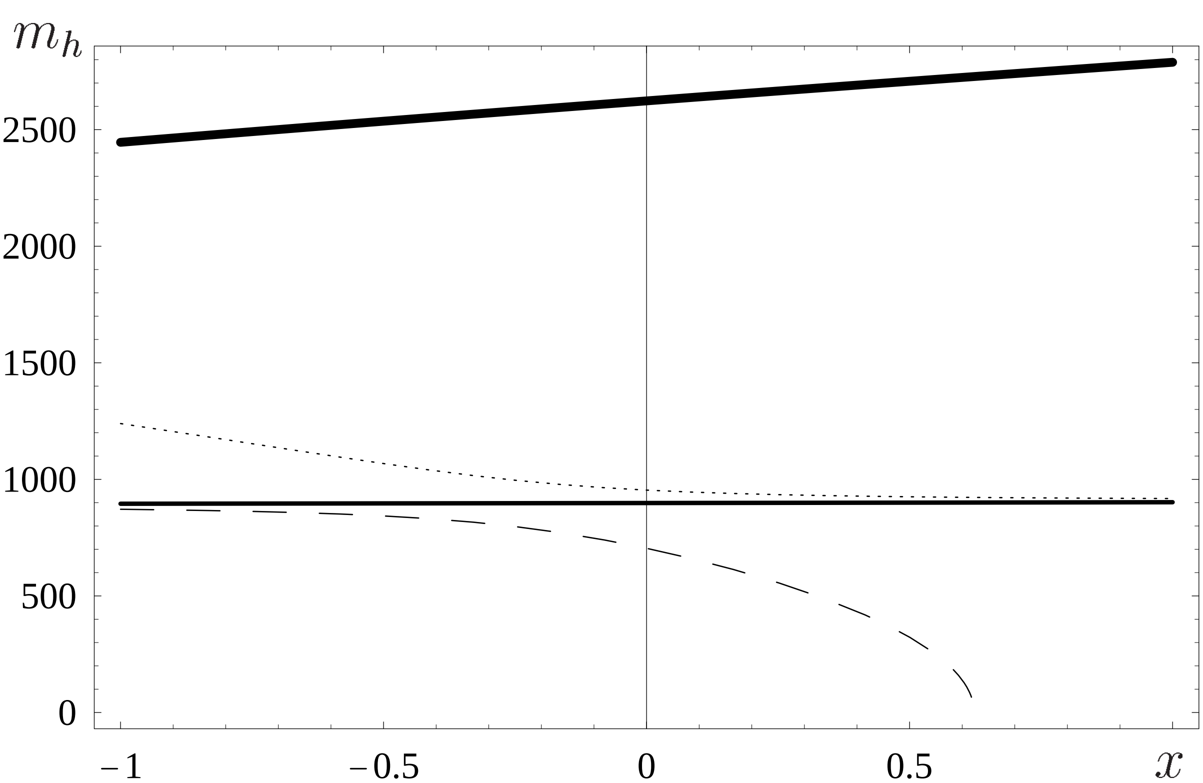
−2.6
−2.4
−2.2
−2
−1.8
z

Fig.4a.



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1500
1000
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0
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0
0.5
x

Fig.4b.

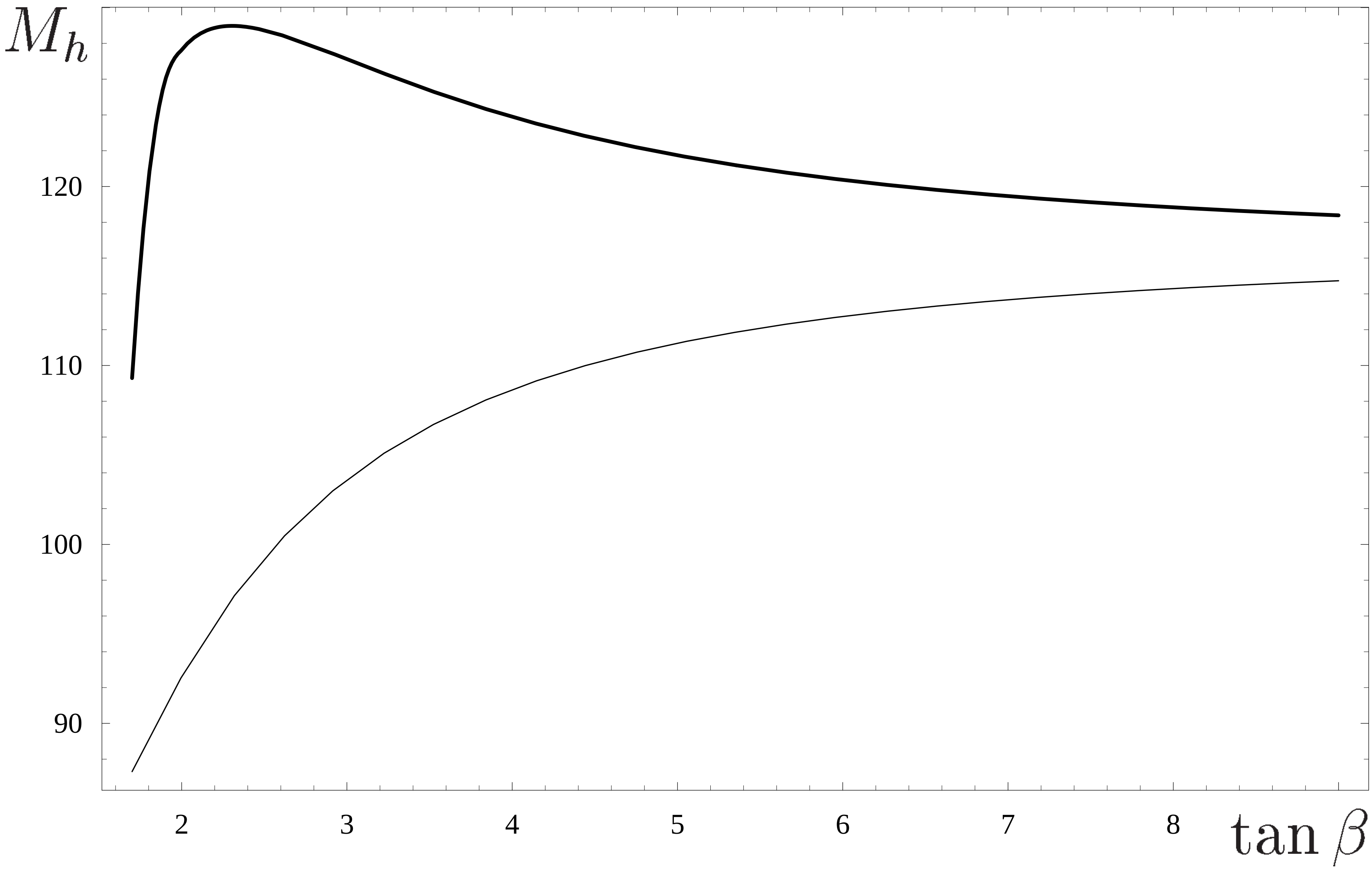

Fig.5.